HOLIDAY HOME WORK

CLASS – XII

SUBJECT - MATHEMATICS

1. ***The following activities from NCERT laboratory manual (Higher Secondary stage) to be done on a practical file.***
   (i) Activity – 3, 4, 9, 11, 12 & 13
2. ***Do the following assignments of chapter 1-5 & 6:***

Chapter 1 :Functions(4/6 marks)

1. Show that an onto function f:{1,2,3} → {1,2,3} is always one-one and vice-versa.

2. Check the injectivity and surjectivity of the function f: N → N defined by

$$f(x) = \begin{cases} \frac{n+1}{2} & , \ n \text{ odd} \\ \frac{n}{2} & , \ n \text{ even} \end{cases}$$

3. Show that if f : A → B and g : B → C are one-one then gof : A → C is also one-one.

4. Show that if f : A → B and g : B → C are onto then gof : A → C is also onto.

5. Let f : W → w be defined as

$$f(x) = \begin{cases} n-1 & , \ n \text{ odd} \\ n+1 & , \ n \text{ even} \end{cases}$$

Show that f is invertible. Find the inverse of f. Where, W is the set of all whole numbers.

6. Show that f : R → {x ∈ R : – 1 < x < 1} defined by $f(x) = \frac{x}{1+|x|}$, x → R is a bijective function.

7. Check the injectivity and surjectivity for the f :, R → R defined by f(x)

$\frac{x}{1+x^2}, \forall x \in R$

8. Let C be the set of complex numbers. Prove that f : C → R given by f(z) = |2|,∀2 ∈ C. is neither one-one nor on to.

9. Let the function f : R → R be defiend by f(x) = cos x ∀ x ∈ R. Show that f is neither one-one nor on to.

10. Let A = [–1, 1]. Then discuss whether f : A → A defined as f(x) = x |x| is one-one, onto or bijective.

11. Classify the following functions as one-one, onto or bijective.

(i) f : N → N defined by f(x) = $x^2 + 1$

(ii) f : Z → Z defined by f(x) = x^3

12. Consider f : R – $f : R - \left\{\frac{-4}{3}\right\} \to R - \left\{\frac{4}{3}\right\}$ given by $f : R = \frac{4x+3}{3x+4}$. Show that f is bijective.

Ans. f^{-1} (x) = $f^{-1}(x) = \frac{4x-3}{4-3x}, f^{-1}(0) = \frac{-3}{4}, x = \frac{11}{10}$

13. Show that f : Q – {3} → Q defined by $f(x) = \frac{2x+3}{x-3}$

14. Let f : [0, ∞) → R be a function defined by f(x) = $9x^2 + 6x - 5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

15. Let f, g : R → R be two functiosn defined as

f(x) = |x| + x and g(x) = |x| – x ∀ x ∈ R.

Then, find fog and gof.

16. If the function f : R → R be defined by f(x) = 2x – 3 and g : R → R by g(x) = $x^3 + 5$, then prove that gof : R → R is a bijective function. Also, verify that $(gof)^{-1} = f^{-1}og^{-1}$.

17. If f : R → R is defined by f(x) = $x^2 - 3x + 2$ write f(f(x))

Ans. $x^4 - 6x^3 + 10x^2 - 3x$

18. Let f : [0, 1] → [0, 1] be defiend as

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$$

Then find (fof) (x)

Ans. x

19. Let f : R → R be defiend by $f(x) = 3x^2 - 5$ and g : R → R by $g(x) = \frac{x}{1 + x^2}$. Then find gof.

Ans. $\frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$

Chapter 2:Inverse Trigonometric Functions (Assignment 2)

1. Find the value of sin $\left[2\cot^{-1}\left(\frac{-5}{12}\right)\right]$ (2 marks)

2. Evaluate cos $\left[\sin^{-1}\frac{1}{4} + \sec^{-1}\frac{4}{3}\right]$ (2 marks)

3. Find the value of $\tan^2(\sec^{-1} 2) + \cot^2(\text{cosec}^{-1} 3)$ (2 marks)

4. If $3\tan^{-1} x + \cot^{-1} x = \frac{\pi}{6}$, then find x. (2 marks)

5. If $\tan^{-1} x + \tan^{-1} y = 4\frac{\pi}{5}$, then find $\cot^{-1} x + \cot^{-1} y$. (2 marks)

6. Simplify $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right), x \in \left[\frac{-3\pi}{4}, \frac{\pi}{4}\right]$ (2 marks)

(4/6 marks)

7. Find the real salutions of the equation:

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

8. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$ and justify why the other value $\frac{4+\sqrt{7}}{3}$ is ignored?

9. If $\alpha \le 2\sin^{-1} x \le \beta$ then find α and β.

10. Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$.

11. Find x for the equation:

$\text{Sin}^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$

12. Show that $2\tan^{-1}\left\{\tan\frac{\alpha}{2}\tan\left(\frac{\pi}{4}-\frac{\beta}{2}\right)\right\} = \tan^{-1}\frac{\sin\alpha\cos\beta}{\cos\alpha+\sin\beta}$

13. Simplify $\tan^{-1} 1 + \tan^{-2} + \tan^{-1} 3$

14. Solve $4\sin^{-1} x = \pi - \cos^{-1} x$

15. Solve $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$.

16. Solve $2 + ax^{-1}(\cos x) = \tan^{-1}(2\text{cosec } x)$

17. Solve $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$

18. Prove that $\cos^{-1}\frac{1}{3} = \sin^{-1}\frac{2\sqrt{2}}{3} = \tan^{-1} 2\sqrt{2}$

19. Solve $\cot^{-1} 2x + \cot^{-1} 3x = \frac{\pi}{4}$

20. Solve $\cos^{-1} x + \sin^{-1}\frac{x}{2} = \frac{\pi}{6}$

21. Express $\sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ in simplest form.

Answers

1. $\frac{-120}{169}$ 2. $\frac{3\sqrt{15}-\sqrt{7}}{16}$

3. 11 4. $x = \frac{-1}{\sqrt{3}}$

5. $\frac{\pi}{5}$ 6. $\tan^{-1}\frac{4}{3} - x$

7. 0, –1 9. $\alpha = 0, \beta = \pi$

10. least value = $\frac{\pi^2}{8}$, greatest value $= \frac{5\pi^2}{4}$

11. 0, $\frac{1}{2}$ 13. π

14. $x = \frac{1}{2}$ 15. –1

16. $x = \frac{\pi}{4}$ 17. $x = \frac{-1}{12}$

19. x = 1 20. 1

21. $\sin^{-1} x - \sin^{-1}\sqrt{x}$

Inverse Trigonometric Functions (Assignment 1)(4/6 marks)

1. Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

2. Prove that $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

3. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

4. Solve $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$, $x > 0$

5. If $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, then find the value of x.

6. Solve $\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$.

7. Show that $\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18 = \cot^{-1} 3$.

8. How many solutions does the equation $\tan^{-1} x - \cot^{-1} x = \tan^{-1}\left[\frac{1}{\sqrt{3}}\right]$ have?

Unique Sol.

9. Solve for x: $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$.

10. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right] = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$.

11. Prove that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

12. Prove that $\cos\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right] = \frac{6}{5\sqrt{13}}$.

13. Prove that $\tan^{-1}\left[\frac{\cos x}{1+\sin x}\right] = \frac{\pi}{4} - \frac{x}{x}, x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

14. Prove that $\sin^{-1}\left(\frac{8}{17}\right) + \sin^{-1}\left(\frac{3}{5}\right) = \cos^{-1}\left(\frac{36}{85}\right)$

15. Solve for x: $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$.

16. Prove that: $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$.

17. Evaluate $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$.

18. If $y = \cot^{-1}\left(\sqrt{\cos x}\right) - \tan^{-1}\left(\sqrt{\cos x}\right)$, then prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$.

(1/2 Mark each)

1. Write the value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$.

2. Find the principal value of $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$.

3. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{-1}{2}\right)$.

4. Write the value of $\cot(\tan^{-1} a + \cot^{-1} a)$.

5. Prove that $\cos\left[\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right] = \frac{6}{5\sqrt{13}}$.

6. Write the principal value of:

(i) $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

(ii) $\cot^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

(iii) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{-1}{2}\right)$

(iv) $\sec^{-1}(-2)$

(v) $\cot^{-1}(-\sqrt{3})$

(vi) $\sin^{-1}\left(\sin\frac{4\pi}{5}\right)$

(vii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

(viii) Write the range of one branch of $\sin^{-1} x$, other than the principal Branch.

Chapter 3,4 :MATRICES AND DETERMINANTS

1/2 MARKERS

1. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ then find A^n.
2. If A = $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ then find the value of α such that A' + A = I.
3. Find a symmetric matrix and a skew symmetric matrix from the matrix $\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix}$.
4. Solve $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix}$ = O .
5. If A is a square matrix such that A^2 = A, then write the value of $(I + A)^2 - 3A$.
6. If $\begin{vmatrix} -3 & y \\ x & -1 \end{vmatrix} = \begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix}$, then find the possible values of x and y, where x , y $\in$ N.
7. Find the value of x for which A = $\begin{bmatrix} x & 1 & 2 \\ 1 & 0 & 3 \\ 5 & -1 & 4 \end{bmatrix}$ is singular.
8. Let A be a square matrix of order 3X3. Write the value of $|2A|$, where $|A|$ =4
9. If A is a square matrix of order 3, $|A| \neq$ and $|3A|$=k $|A|$, then write the value of k.
10. If A and B are the square matrices of order3, such that $|A|$= -1 and $|B|$ = 3, then find the value of $|7AB|$.
11. If A and B are the square matrices of same order, such that $|A|$= 6 and AB = I, then write the value of $|B|$.
12. If A is a skew symmetric matrix of order 3, write the value of $|A|$.
13. If A is an invertible matrix of order 3, and $|adjA|$= 64, then find the value of $|A|$.
14. If A is a square matrix such that A(adjA) = 8 I, then find $|A|$.
15. If A is a matrix of order 3, such that A (adj A) = $\begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix}$, then find $|adjA|$.
16. If A is a non- singular square matrix such that $|A|$= 5, then find $|A^{-1}|$.

17. If area of Δ is 35 sq units with vertices (2,-6), (5,4) and (k, 4), then find k.

18. For what value of and y, is the matrix A = $\begin{bmatrix} 0 & 1 & y \\ -1 & x & -4 \\ -3 & 4 & 0 \end{bmatrix}$ is a skew symmetric matrix.

19. If A is a square matrix and $|A|$ = 2, then write the value of $|AA^1|$.

20. If A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and A^2 = kA, then write the value of k.

21. For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse?

22. If $\begin{bmatrix} a+10 & b^2+2b \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3a+4 & 3 \\ 0 & b^2-5b \end{bmatrix}$, then find a and b.

23. If A is a matrix of order m X n and B is a matrix such that $AB^1 = B^1A$ are both defined, then find the order of B.

24. Find the maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$.

MATRICES AND DETERMINANTS

4/6 MARKERS

1. The monthly incomes of Aryan and Babban are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs. 15000 per month, find their monthly incomes using matrix method.
   [Ans : x =30000, y = 15000]

2. A farmer posses 30 acre cultivated land that must be cultivated in two different mode of cultivations organic and inorganic. The yield for organic and inorganic system of cultivations is 11 quintals/acre and 14 quintals/acre respectively. Using matrix method, determine how to divide 30 acre land among two mode of cultivation to obtained yield 390 quintals.
   (Ans : 10 , 20)

3. Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality(y) and obedience(z). School A decided to award a total of 11000 for three values to 5,4, and 3 students respectively, while school B decided to award 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to 2700, then

i) Represent the above situation by a **matrix equation** and form **linear equations** using matrix multiplication.

ii) Is it possible to solve the system of equations so obtained using matrices?

4. A typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices find the charges of typing one English and one Hindi page separately. **[Ans : Hindi = Rs. 15, English = Rs 10]**

5. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12 % interest. The trust received Rs. 2800 as interest. However, if trust had interchanged money in bonds, they would have got Rs. 100 less as the interest. Using matrix method, find the amount invested by the trust.

6. A shopkeeper has three varieties of pens ‘A’, ‘B’ and ‘C’. Meenu purchased one pen of each variety for a total of Rs.21. Jeevan purchased 4 pens of

variety 'A', 3 pens of variety 'B' and 2 pens of variety 'C' for Rs. 60. While Shikha purchased 6 pens of variety 'A', 2 pens of variety 'B' and 3 pens of variety 'C' for Rs. 70. Using matrix method, find the cost of each variety of pen.

7. On her birthday, Seema decided to donate some money to children of an orphanage home. If there are 8 children less, every one would have got 10 Rs more. However, if there were 16 children more, everyone would have got Rs. 10 less. Using matrix method , find the number of children and the amount distributed by Seema.
8. A man wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50m and breadth is increased by 50 m, then its area will remain same, but if its length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m sq. Using matrices find the dimensions of the plot.

   [Ans : 200m, 150m]

9. Let A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$, then compute AB. Hence solve the following system of equations 2x + y = 4, 3x + 2y = 1.
10. An amount of Rs. 5000 is put into 3 investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs. 358. If the combined income from the first two investments is Rs. 70 more than the income from the third. Find the amount of each investment by matrix method.
11. The managing committee of a residential colony decided to award some of its members (say x) for honesty, some (say y) for helping others and some others (say z) for supervising the workers to keep the colony neat and clean. The sum of all the awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the number of the awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method , find the number of awardees of each category. **[Ans : x =3, y = 4, z = 5]**

12.If A = $\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find x and y such that A^2 - xA +yI = O. hence evaluate A^{-1}.

13.Without expanding, prove that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$.

14.Solve for x:

i) $\begin{vmatrix} 15-2x & 11-3x & 7-x \\ 11 & 17 & 14 \\ 10 & 16 & 13 \end{vmatrix} = 0$

ii) $\begin{vmatrix} 1 & 1 & x \\ \beta+1 & \beta+1 & \beta+x \\ 3 & x+1 & x+2 \end{vmatrix} = 0$, $\beta \neq 0$ [x = 1, 2]

iii) $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$ [A: x = 4]

iv) $\begin{vmatrix} 1 & x & x^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = 0$ where, b is different than c. [A: x= b , c, - (b+c)]

15.If f(x) = $\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, using properties of determinant find the value of $f(2x) - f(x)$.

16.If x,y,z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then show that $1 + xyz = 0$

17.If a, b and c are in A.P then find the value of $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$

18.If a, b and c are the pth , qth and rth terms respectively of a G.P, then prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

19. If a, b, c are positive and unequal, show that the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is always negative.

20. If A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, then find A^{-1} by elementary transformations and hence solve the following system of equations:

$3x + 4y + 7z = 14,\ \ 2x - y + 3z = 4,\ \ \ x + 2y - 3z = 0$

[A : x = y = z = 1]

21. Given two matrices A and B, A = $\begin{bmatrix} 1 & -2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$. Find AB and use the result to solve system of equations: x – 2y + 3z = 6,
X + 4y + z = 12, x – 3y + 2z = 1**[Ans : AB = - 8I, x=1,y=2,z=3]**

(Q 12 – Q 27) - Using the properties of the determinants, prove that:

22. $\begin{vmatrix} 1 & a^2 + bc & a^3 \\ 1 & b^2 + ca & b^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a^2 + b^2 + c^2)$.

23. $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha + \delta) \\ \sin\beta & \cos\beta & \cos(\beta + \delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma + \delta) \end{vmatrix} = 0$

24. $\begin{vmatrix} b^2c^2 & bc & b + c \\ c^2a^2 & ca & c + a \\ a^2b^2 & ab & a + b \end{vmatrix} = 0$

25. $\begin{vmatrix} 1 + a & 1 & 1 \\ 1 & 1 + b & 1 \\ 1 & 1 & 1 + c \end{vmatrix} = ab + bc + ca + abc$

26. $\begin{vmatrix} a & a + b & a + b + c \\ 2a & 3a + 2b & 4a + 3b + 2c \\ 3a & 6a + 3b & 10a + 6b + 3c \end{vmatrix} = a^3$

27. $\begin{vmatrix} a^2 & bc & c^2+ac \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

28. $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

29. $\begin{vmatrix} a & a+c & a-b \\ b-c & b & b+c \\ c+b & c-a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

30. $\begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

31. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

32. $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

33. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

34. $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc-a^3-b^3-c^3$

35. $\begin{vmatrix} \alpha & \alpha^2 & \beta+\gamma \\ \beta & \beta^2 & \gamma+\alpha \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix} = (\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)(\alpha+\beta+\gamma)$

36. $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

37. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

38. $\begin{vmatrix} \frac{(x+y)^2}{x} & x & x \\ y & \frac{(z+x)^2}{y} & y \\ z & z & \frac{(x+y)^2}{z} \end{vmatrix} = 2(x+y+z)^3$

39. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

40. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$

41.If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0$ then show that either p, q, r are in G.P i.e, $q^2 = rp$ or α is the root of $px^2+2qx+r=0$.

42. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ a & a^2 & 1 \\ a^2 & a & 1 \end{vmatrix} = -4$ then find the value of $\begin{vmatrix} a^3-1 & 0 & a-a^4 \\ 0 & a-a^4 & a^3-1 \\ a-a^4 & a^3-1 & 0 \end{vmatrix}$

Chapter 5:Continuity and Differentiability

4 marks

1. The function f is defined as

$$f(x) = \begin{cases} x^2 + ax + b & 0 \le x < 2 \\ 3x + 2 & 2 \le x \le 4 \\ 2ax + 5b & 4 < x \le 8 \end{cases}$$

If function is continuous on [0, 8], find the values of a and b.

2. Discuss the continuity of the function f given by

$f(x) = |x - 1| + |x - 2|$

3. Examine the continuity of the function f given by

$$f(t) = \begin{cases} \dfrac{\cos t}{\frac{\pi}{2} - t}; & t \ne \dfrac{\pi}{2} \\ 1; & t = \dfrac{\pi}{2} \end{cases} \text{ at } t = \frac{\pi}{2}$$

4. Let $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & x > 0 \end{cases}$ Determine the value of a so that f is continuous at $x = 0$. (NCERT Exemplar CBSE 2010, 2012)

5. Determine the values of a, b, c for which the function given by

$$f(x) = \begin{cases} \dfrac{\sin(a + 1)x + \sin x}{x} & x < 0 \\ c & x = 0 \\ \dfrac{\sqrt{x + bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

is continuous at $x = 0$.

6. (i) If $f(x) = \begin{cases} \dfrac{x - 4}{|x - 4|} + a & x < 4 \\ a + b & x = 4 \\ \dfrac{x - 4}{|x - 4|} + b & x > 4 \end{cases}$ is continuous at $x = 4$, find a and b.

(ii) Determine if f defined by $f(x) = \begin{cases} x^2 \sin \frac{1}{x}; & x \neq 0 \\ 0; & x = 0 \end{cases}$ is a continuous function?

7. If $f(x) = \begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2 + 1} - 1}; & x \neq 0 \\ K; & x = 0 \end{cases}$ is continuous at x = 0, find K.

8. Find the value of 'a' for which the function f defined by

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x + 1); & x \leq 0 \\ \dfrac{\tan x - \sin x}{x^3}; & x > 0 \end{cases}$$ should be continuous at x = 0. (CBSE 2011)

9. For what value of K is the following function continuous at x = 2

$$f(x) = \begin{cases} 2x + 1; & x < 2 \\ K; & x = 2 \\ 3x - 1; & x > 2 \end{cases}$$ (CBSE 2008)

10. $$f(x) = \begin{cases} \dfrac{1 - \sin^3 x}{3 \cos^2 x}; & x < \frac{\pi}{2} \\ a; & x = \frac{\pi}{2} \\ \dfrac{b(1 - \sin x)}{(\pi - 2x)^2}; & x < \frac{\pi}{2} \end{cases}$$

If f is continuous at $x = \frac{\pi}{2}$, find a and b.

11. If the function f defined below is continuous at x = 0, find the value of K:

$$f(x) = \begin{cases} \dfrac{1 - \cos 2x}{2x^2} & x < 0 \\ K & x = 0 \\ \dfrac{x}{|x|} & x > 0 \end{cases}$$ (CBSE 2010)

12. Show that the function defined by g(x) = x – [x] is discontinuous at all integral points. Here [x] denotes the greatest integer less than or equal to x.

13. In the following, determine the value of constant involved in the definition so that the given function is cotinuous:

(i) $f(x) = \frac{\sqrt{1+px} - \sqrt{1-px}}{x}$ $-1 \le x < 0$

$\frac{2x+1}{x-2}$ $0 \le x \le 1$

(ii) $f(x) = \begin{cases} \frac{K\cos x}{\pi - 2x} & x < \frac{\pi}{2} \\ 3 & x = \frac{\pi}{2} \\ \frac{3\tan 2x}{2x - \pi} & x > \frac{\pi}{2} \end{cases}$ (CBSE 2010)

14. $f(x) = \frac{\sqrt{2}\cos x - 1}{\cot x - 1}$ $x \ne \frac{\pi}{4}$

Find the value of $f\left(\frac{\pi}{4}\right)$ so that f becomes continuous at $x = \frac{\pi}{4}$.

15. Show that the function f given by

$f(x) = \begin{cases} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \ne 0 \\ 0 & x = 0 \end{cases}$

is discontinuous at x = 0.

16. Let $f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16} \end{cases}$ if $x \ne 2$.

For what value of k, f is continuous at x = 2.

17. Prove that [x] is continuous at all points except at integers.

18. Check the differentiability of cos |x| on R.

19. If $f(x) = \frac{1}{x-1}$. Find the points of discontinuity of f[f(x)]

Assignment 1(Answers)

1. a = 3, b = – 2

4. a = 8

5. $a = \frac{-3}{2}, b \in R - \{0\}, c = \frac{1}{2}$

6. (i) a = 1, b = – 1

7. k = – 4

8. $a = \frac{1}{2}$

9. K = 5

10. $a = \frac{1}{2}$ b = 4

11. K = 1

13. (i) $p = -\frac{1}{2}$ (ii) K = 6

14. $\frac{1}{2}$

16. $K = \frac{1}{2}$

18. Continuous every where

19. 1 & 2

Assignment – 2

Differentiation

1. Diff. the following fns w.r.t. x(1/2 marks)

(i) $\sin(\cos x^2)$ (ii) $\sec(\tan\sqrt{x})$

(iii) $\cos x^3.\sin^2 x^5$ (iv) $2\sqrt{\cot x^2}$

(v) $\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ (vi) $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$

(vii) $\cos^{-1}(\tan^2\sqrt{4x+5})$ (viii) $\sqrt{e^{\sqrt{\cos\sqrt{x}}}}$

(ix) $\cos(\log x + e^x + \sin x)$

(x) $\sqrt{\frac{(x-3)(x^2+4)}{(3x^2+4x+5)(x-4)}}$

(xi) $\cos x\cos 2x.\cos 3x$

2. Find $\frac{dy}{dx}$ if: (4/6 marks)

(i) $y = (\log x)^{\cos x} + x^x - 2^{\sin x}$

(ii) $y = \left(x+\frac{1}{x}\right)^x + _x\left(1+\frac{1}{x}\right)$

(iii) $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

(iv) $y^x = x^y$

(v) $xy = e^{x-y}$

(vi) $(\cos x)^y = (\cos y)^x$

(vii) $y = x^{x^x} + (x^x)^x$

3. If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{\cos^{-1}t}}$ show that $\frac{dy}{dx} = \frac{-y}{x}$

Also, find $\frac{d^2y}{dx^2}$.

4. Diff. $\sin^{-1}\frac{2^{x+1}}{1+4^x}$ w.r.t. $\tan^{-1}\frac{\sin x}{1+\cos x}$

5. Find $\frac{dy}{dx}$ if $y = a^{t+\frac{1}{t}}, x = \left(t + \frac{1}{t}\right)^a$

6. Diff. $\tan^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

w.r.t $(\log x)^{\log x}$

7. Diff. $x^x + a^x + x^a + a^a$

w.r.t. x

8. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ Prove that $\frac{dy}{dx} = \frac{1}{(1+x)^2}$

9. If $y^x = e^{y-x}$ Prove that $\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y}$

10. If $x = \tan\left(\frac{1}{a}\log y\right)$

Prove that $(1 + x^2) y_2 + (2x - a) y_1 = 0$

11. If $x = a\cos\theta + b\sin\theta$

$Y = a\sin\theta - b\cos\theta$ then

Prove that $y^2 y_2 \ xy_1 + y = 0$

12. If $x = \sin t$, $y = \sin pt$ then prove that $(1 - x^2) y_2 - xy_1 + b^2y = 0$

13. If $y = \sqrt{x+1} - \sqrt{x-1}$ Prove that $(x^2-1)y_2 + xy_1 = \frac{y}{4} = 0$

14. Diff. $\cos^{-1}\left[x\sqrt{1-x} + \sqrt{x}\sqrt{1-x^2}\right]$ w.r.t. x.

15. Diff. $\sin^{-1}\left[\dfrac{6x - 4\sqrt{1-4x^2}}{5}\right]$ w.r.t. x

16. Diff. $\tan^{-1}\left(\dfrac{x}{1+6x^2}\right)$ w.r.t. $\tan^{-1}\left(\dfrac{4\sqrt{x}}{1-4x}\right)$

17. If $y = \sec^{-1}\left[\dfrac{\sqrt{x}+1}{\sqrt{x}-1}\right] + \sin^{-1}\left[\dfrac{\sqrt{x}+1}{\sqrt{x}+1}\right]$

Then find $\dfrac{dy}{dx}$

18. Diff. $y = f\left(\dfrac{2x-1}{x^2+1}\right)$ if $f^1(x) = \sin(x^2)$

19. Diff. $\tan^{-1}\left[\sqrt{1+b^2x^2} - bx\right]$

w.r.t. $\tan^{-1}$ [cosec x + cot x]

20. If $x^y = e^{x-y}$ then find $\dfrac{dy}{dx}$

21. If $y = x^{x^{x^{\cdots\infty}}}$ then prove that

$Y^1 = \dfrac{y^2}{x(1-\log y)}$

22. If $\begin{array}{l} x = e^\theta(\sin\theta + \cos\theta) \\ y = e^\theta(\sin\theta - \cos\theta) \end{array}$ find $\dfrac{d^2y}{dx^2}$

23. Find $\dfrac{dy}{dx}$ of each of the following functions:

(i) $x = e^\theta\left(a + \dfrac{1}{\theta}\right), y = e^{-\theta}\left(\theta - \dfrac{1}{\theta}\right)$

(ii) $\sin x = \dfrac{1}{\sqrt{1+t^2}}$ and $\cot y = \dfrac{2t}{1-t^2}$

(iii) $x = \left(\theta + \frac{1}{\theta}\right)^{a}, y = a^{\theta + \frac{1}{\theta}}$

24. If $y = \left(x + \sqrt{1 + x^2}\right)^n$ then show that $(1 + x^2)\, y_2 + xy_1 + x^2y = 0$

25. If $y = x \log\left(\frac{x}{a + bx}\right)$ m then prove that $x^3y_2 = (xy_1 - y)^2$

26. Diff. $\sin^{-1}\left[\frac{2^{x+1}3^x}{1 + (36)^x}\right]$ w.r.t. x

27. If sin y = x sin (a + y) then prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$

28. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$ then prove that $y^1 = \frac{-y \log x}{x \log y}$

29. If $x^m y^n = (x + y)^{m+n}$ Prove that (i) $y^1 = \frac{y}{x}$ (ii) $y^{11} = 0$

30. If $y = \tan^{-1}\frac{5x}{1 - 6x^2}, \frac{-1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$ then prove that $y^1 = \frac{2}{1 + 4x^2} + \frac{3}{1 + 9x^2}$

31. If $e^x + e^y = e^{x+y}$

Prove that $y^1 = -e^{y-x}$

32. For what choice of a and b is the function $f(x) = \begin{cases} x^2 & x \le c \\ ax + b & x > c \end{cases}$ is differentiable at x = c.

33. Discuss the differentiability of $|x - 1| + |x - 2|$

34. If $\sqrt{1 - x^6} + \sqrt{1 - y^6} = a(x^3 - y^3)$ prove that $y^1 = \frac{x^2}{y^2}\sqrt{\frac{1 - y^y}{1 - x^6}}$

35. If $y = \{\log_{\cos x} \sin x\}\{\log_{\sin x} \cos x\}^{-1} + \sin^{-1}\frac{2x}{1 + x^2}$

Find $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$

36. If $x = a(\cos\theta + \sin\theta)$

$Y = a(\sin\theta - \theta\cos\theta)$

Find $\frac{d^2x}{d\theta^2}, \frac{d^2y}{d\theta^2}$ and $\frac{d^2y}{dx^2}$

37. $f(x) = \sqrt{1+x^2}, g(x) = \frac{x+1}{x^2+1}$ and $h(x) = 2x - 3$ then find $f^1[h^1\{g^1(x)\}]$

38. If $f(x) = \begin{vmatrix} x+a^2 & ab & ac \\ ab & x+b^2 & bc \\ ac & bc & x+c^2 \end{vmatrix}$

Prove that $f^1(x) = 3x^2 + 2x(a^2 + b^2 + c^2)$

Assignment – 3

Mean Value Theorems (4 Marks)

1. Discuss the applicability of Rolle's in for the following functions on the indicated intervals:

   (i) $f(x) = 32 + (x - 2)^{2/3}$ on [1, 3]

   (ii) $f(x) = x^2 + 1 \quad 0 \leq x \leq 1$

   $3 - x \quad 1 < x \leq 2$

   3. $f(x) = \sqrt{4 - x^2}$ on [−2, 2]

   (iv) $f(x) = (x - a)^m (x - b)^n$ on [a, b]

   Where m and n are positive integers

   (v) $f(x) = \sin x - \sin 2x$ on $[0, \pi)$

2. Find the point on the curve $y = \cos x - 1$ $x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ at which the tangent is parallel to the x-axis.

3. Verify Lagrange's mean value th for the following functions on the indicated intervals:

   (i) $f(x) = x - 2\sin x$ on $[-\pi, \pi]$

   (ii) $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

   (iii) $f(x) = \frac{1}{4x - 1}$, $1 \leq x \leq 4$

4. Find a point on the curve $y = x^3 + 1$ where the tangent is parallel to the chord joining (1, 2) and (3, 28).

Chapter 5 : APPLICATIONS OF DERIVATIVES

RATE OF CHANGE

1. Water is dripping out at a steady rate of 1 cu cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4cm, find the rate of decrease of slant height, where the vertical angle of the conical vessel is $\frac{\pi}{6}$. [A : $\frac{1}{2\sqrt{3}\pi} cm/s$]
2. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.
3. If the area of the circle increases at a uniform rate, then prove that perimeter varies inversely as the radius.
4. A kite is moving horizontally at a height of 151.5 meters. If the speed of kite is 10 m/s, how fast is the string being let out; when the kite is 250 m away from the boy who is flying the kite? The height of boy is 1.5 m. [A: 8 m/s]
5. A man, 2m tall, walks at the rate of $1\frac{2}{3}$ m/s towards a street light which is $5\frac{1}{3}$ m above ground. At what rate is the tip of his shadow moving? At what rate is the length of the shadow changing when he is $3\frac{1}{3}$ m from the base of the light?

   [A: $2\frac{2}{3} m/s$ towards light, $-1 m/s$]
6. x and y are the sides of two squares such that $y = x - x^2$. Find the rate of change of the area of second square with respect to the area of first square.

   [A: $2x^3 - 3x + 1$]
7. A ladder, 5 meter long, standing on a horizontal floor, leans against a vertical wall. If the top of the ladder slides downwards at the rate of 10 cm/sec, then the rate at which the angle between the floor and the ladder is decreasing when lower end of ladder is 2 meters from the wall is? [A: 1/20 radians/sec]

8. For the curve y = 5x – 2x^3 , if x increases at the rate of 2 units/sec, then how fast is the slope of the curve changing when x = 3. [Ans: -72 units/sec]
9. The sides of an equilateral triangle are increasing at the rate of 2 cm/ sec. find the rate at which the area increases, when side is 10 cm. [A: $10\sqrt{3}\ cm^2/s$]
10. Water is running into a conical vessel, 15 cm deep and 5 cm in radius, at the rate of 0.1 cu cm/sec. when the water is 6 cm deep, find at what rate is
    i) The water level rising?
    [A: $\frac{1}{40\pi}cm/s$]
    ii) The water surface area increasing?
    [A: $\frac{1}{30}cm^2/s$]
    iii) The wetted surface of the vessel increasing?
    [A: $\frac{\sqrt{10}}{30}cm^2/s$]

INCREASING/ DECRESING FUNCTIONS
4/6 MARKERS

1. Find the intervals on which the following functions are I) strictly increasing or strictly decreasing II) increasing or decreasing:
    i) $f(x) = -2x^3 - 9x^2 - 12x + 1$
    ii) $f(x) = \frac{x}{2} + \frac{2}{x}(-\infty, 0) \cup (0, \infty)$
    iii) $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$
    iv) $f(x) = 5x^{3/2} - 3x^{5/2} \quad on\ (0, \infty)$
    v) $f(x) = \cos^2 x$ on $[0, \frac{\pi}{2}]$
    vi) $f(x) = \sin\left(2x + \frac{\pi}{4}\right)$ on $\left(\frac{3\pi}{8} + \frac{5\pi}{8}\right)$
    vii) $f(x) = \sin^4 x + \cos^4 x$ on $[0, \frac{\pi}{2}]$
2. Prove that the function f given by f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$

3. Find the intervals in which the function $f(x) = \frac{4\sin x}{2+\cos x} - x\ ;\ 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing

HOTS:

1. Find the intervals in which the following functions are I) strictly increasing or strictly decreasing II) increasing or decreasing:

i) $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ on $[0, \pi]$

ii) $f(x) = \tan^{-1} x - x$

iii) $f(x) = \tan^{-1}(\sin x + \cos x)$ on $\left(0, \frac{\pi}{4}\right)$

2. Show that $f(x) = 2x + \cot^{-1} x + \log\left(\sqrt{1+x^2} - x\right)$ is increasing on ***R.***

3. Show that for $a \ge 1$, $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ is decreasing on R