Deep Public School MATHS ASSIGNMENT X (2017-18)

35. $\left(\frac{3\cos 43^\circ}{\sin 47^\circ}\right)^2 - \frac{\cos 37^\circ \operatorname{cosec} 53^\circ}{\tan 5^\circ \tan 25^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ}$

36. If $\sec 2A = \operatorname{cosec}(A - 42^\circ)$ where $2A$ is an acute angle, find the value of A.

37. If $\sin(A-B) = 0, \cos(A+B) = 0, 0^\circ < A+B \le 90^\circ$, find A and B.

38. If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 0, 0^\circ < A+B \le 90^\circ$, find $\sin(A+B)$ and $\cos(A-B)$.

39. Simplify: $(1+\tan^2\theta)(1-\sin\theta)(1+\sin\theta)$.

40. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

41. Given that $\alpha + \beta = 90^\circ$, show that $\sqrt{\cos\alpha \operatorname{cosec}\beta - \cos\alpha \sin\beta} = \sin\alpha$.

42. If $\tan\theta = \frac{a}{b}$, prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$.

43. If $\sec\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$.

44. If $\theta = 30^\circ$, verify that $\tan^2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

45. If $\theta = 30^\circ$, verify that $\cos^2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

46. If $A = 30^\circ$ and $B = 60^\circ$, verify that $\cos(A+B) = \cos A\cos B - \sin A\sin B$.

47. If $A = 30^\circ$ and $B = 60^\circ$, verify that $\sin(A+B) = \sin A\cos B + \cos A\sin B$.

48. If $\cot\theta = \frac{15}{8}$, then evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$.

49. If $\sec\theta = x + \frac{1}{x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$.

50. If $\sqrt{3}\tan\theta = 3\sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$.

51. If $\operatorname{cosec}\theta = \frac{13}{12}$, find the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$.

52. If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$, find $1 + \tan\theta\cos\theta$.

53. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Find the value of x : (54–55)

54. $\sqrt{3}\sin x = \cos x$

55. $\tan x = \sin 45^\circ\cos 45^\circ + \sin 30^\circ$

Prove the following trigonometric identities: (56–63)

56. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

57. $\tan\theta + \tan(90^\circ - \theta) = \sec\theta\sec(90^\circ - \theta)$

58. $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

59. $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = 1 + \tan A + \cot A$

60. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

61. $\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}$

62. $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

63. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **(4 marks)**

Long Answer Questions

64. If $a\sin\theta + \cos\theta = c$, then prove that $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$.

65. Given that $\sin\theta + 2\cos\theta = 1$, then prove that $2\sin\theta - \cos\theta = 2$.

66. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\frac{1}{2}$.

67. If $\sec\theta + \tan\theta = p$, show that $\frac{p^2 - 1}{p^2 + 1} = \sin\theta$.

68. If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

Prove that the following identities (69–80)

69. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

70. $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

71. $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

72. $\frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

73. $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1 + \sin\theta}{\cos\theta}$

74. $(1 + \cot\theta + \tan\theta)(\sin\theta - \cos\theta) = \frac{\sec\theta}{\operatorname{cosec}^2\theta} - \frac{\operatorname{cosec}\theta}{\sec^2\theta}$

75. $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{2\sin^2 A - 1}$

76. $(\sin\theta - \sec\theta)^2 + (\cos\theta - \operatorname{cosec}\theta)^2 = (1 - \sec\theta\operatorname{cosec}\theta)^2$

77. $\frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \cot\theta$

78. $\frac{\operatorname{cosec} A}{\operatorname{cosec} A - 1} + \frac{\operatorname{cosec} A}{\operatorname{cosec} A + 1} = 2\sec^2 A$

79. $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\operatorname{cosec} A\cot A$

80. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = \frac{1}{\tan\theta + \cot\theta}$

81. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

82. $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}} = 2\operatorname{cosec}\theta$

83. $(\sec A - \operatorname{cosec} A)(1 + \tan A + \cot A) = \tan A\sec A - \cot A\operatorname{cosec} A$.

84. $\sin A(1 + \tan\theta)^2 + \cos A(1 + \cot A) = \sec A + \operatorname{cosec} A$

85. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$