

IMPORTANT SNAPS BY TEAM PIS CLASS- X TH

Subject: MATHEMATICS
Teacher: Ms. PRIYA KHATRI
(chapter 8 to 15)

HIGHLIGHTS OF CHAPTER 8

INTRODUCTION TO TRIGONOMETRY

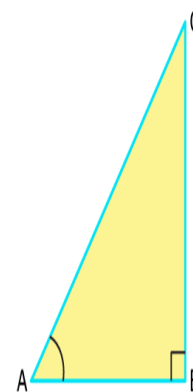
- ◉ **Trigonometric Ratios**

- ◉ For the right $\triangle ABC$, right-angled at $\angle B$, the trigonometric ratios of the $\angle A$ are as follows:

- ◉ $\sin A = \text{opposite side} / \text{hypotenuse} = BC / AC$
- ◉ $\cos A = \text{adjacent side} / \text{hypotenuse} = AB / AC$
- ◉ $\tan A = \text{opposite side} / \text{adjacent side} = BC / AB$
- ◉ $\operatorname{cosec} A = \text{hypotenuse} / \text{opposite side} = AC / BC$
- ◉ $\sec A = \text{hypotenuse} / \text{adjacent side} = AC / AB$
- ◉ $\cot A = \text{adjacent side} / \text{opposite side} = AB / BC$

Relation between Trigonometric Ratios

- ◉ $\operatorname{cosec} \theta = 1 / \sin \theta$
- ◉ $\sec \theta = 1 / \cos \theta$
- ◉ $\tan \theta = \sin \theta / \cos \theta$
- ◉ $\cot \theta = \cos \theta / \sin \theta = 1 / \tan \theta$



- ◉ **Trigonometric Ratios of Specific Angles**
- ◉ Range of Trigonometric Ratios from 0 to 90 degrees
- ◉ For $0^\circ \leq \theta \leq 90^\circ$,
- ◉ $0 \leq \sin \theta \leq 1$
- ◉ $0 \leq \cos \theta \leq 1$
- ◉ $0 \leq \tan \theta < \infty$
- ◉ $1 \leq \sec \theta < \infty$
- ◉ $0 \leq \cot \theta < \infty$
- ◉ $1 \leq \operatorname{cosec} \theta < \infty$
- ◉ $\tan \theta$ and $\sec \theta$ are not defined at 90° .
 $\cot \theta$ and $\operatorname{cosec} \theta$ are not defined at 0° .
- ◉ **Variation of trigonometric ratios from 0 to 90 degrees**
- ◉ As θ increases from 0° to 90°
- ◉ $\sin \theta$ increases from 0 to 1
- ◉ $\cos \theta$ decreases from 1 to 0
- ◉ $\tan \theta$ increases from 0 to ∞
- ◉ $\operatorname{cosec} \theta$ decreases from ∞ to 1
- ◉ $\sec \theta$ increases from 1 to ∞
- ◉ $\cot \theta$ decreases from ∞ to 0

Standard values of Trigonometric ratios

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos A$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan A$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec} A$	not defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec A$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	not defined
$\cot A$	not defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

Trigonometric Ratios of Complementary Angles

- ◉ Complementary Trigonometric ratios
- ◉ If θ is an acute angle, its complementary angle is $90^\circ - \theta$. The following relations hold true for trigonometric ratios of complementary angles.
- ◉ $\sin (90^\circ - \theta) = \cos \theta$
- ◉ $\cos (90^\circ - \theta) = \sin \theta$
- ◉ $\tan (90^\circ - \theta) = \cot \theta$
- ◉ $\cot (90^\circ - \theta) = \tan \theta$
- ◉ $\operatorname{cosec} (90^\circ - \theta) = \sec \theta$
- ◉ $\sec (90^\circ - \theta) = \operatorname{cosec} \theta$

Trigonometric Identities

- ◉ $\sin^2 \theta + \cos^2 \theta = 1$
- ◉ $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$
- ◉ $1 + \tan^2 \theta = \sec^2 \theta$

EXTRA QUESTIONS

- Question 1: If $\tan (A + B) = \sqrt{3}$ and $\tan (A - B) = 1/\sqrt{3}$, $0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.
- Solution: Given,
- $\tan (A + B) = \sqrt{3}$
- As we know, $\tan 60^\circ = \sqrt{3}$
- Thus, we can write;
- $\Rightarrow \tan (A + B) = \tan 60^\circ$
- $\Rightarrow (A + B) = 60^\circ \dots\dots (i)$
- Now again given;
- $\tan (A - B) = 1/\sqrt{3}$
- Since, $\tan 30^\circ = 1/\sqrt{3}$
- Thus, we can write;
- $\Rightarrow \tan (A - B) = \tan 30^\circ$
- $\Rightarrow (A - B) = 30^\circ \dots\dots (ii)$
- Adding the equation (i) and (ii), we get;
- $A + B + A - B = 60^\circ + 30^\circ$
- $2A = 90^\circ$
- $A = 45^\circ$
- Now, put the value of A in eq. (i) to find the value of B;
- $45^\circ + B = 60^\circ$
- $B = 60^\circ - 45^\circ$
- $B = 15^\circ$
- Therefore $A = 45^\circ$ and $B = 15^\circ$

- Question 2: Prove the identities:
- (i) $\sqrt{1 + \sin A / 1 - \sin A} = \sec A + \tan A$
- Solution: (i) Given: $\sqrt{1 + \sin A / 1 - \sin A} = \sec A + \tan A$

$$\text{L.H.S} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

First divide the numerator and denominator of L.H.S. by $\cos A$,

$$= \sqrt{\frac{\frac{1}{\cos A} + \frac{\sin A}{\cos A}}{\frac{1}{\cos A} - \frac{\sin A}{\cos A}}}$$

We know that $1/\cos A = \sec A$ and $\sin A / \cos A = \tan A$ and it becomes,

$$= \sqrt{\frac{\sec A + \tan A}{\sec A - \tan A}}$$

Now using rationalization, we get

$$= \sqrt{\frac{\sec A + \tan A}{\sec A - \tan A}} \times \sqrt{\frac{\sec A + \tan A}{\sec A + \tan A}}$$

$$= \sqrt{\frac{(\sec A + \tan A)^2}{\sec^2 A - \tan^2 A}}$$

$$= (\sec A + \tan A)/1$$

$$= \sec A + \tan A = \text{R.H.S}$$

Hence proved

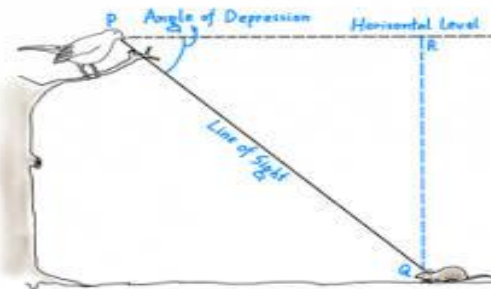
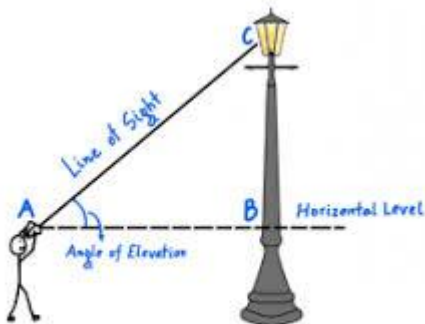
HIGHLIGHTS OF CHAPTER 9

APPLICATION OF TRIGONOMETRY

Heights and Distances

Horizontal Level and Line of Sight

- Line of sight and horizontal level **Line of sight** is the line drawn from the eye of the observer to the point on the object viewed by the observer.
- Horizontal level** is the horizontal line through the eye of the observer.
- Angle of elevation
- The **angle of elevation** is relevant for objects above horizontal level. It is the **angle** formed by the **line of sight** with the **horizontal level**.
- Angle of depression
- The **angle of depression** is relevant for objects below horizontal level. It is the **angle** formed by the **line of sight** with the **horizontal level**.



◉ Calculating Heights and Distances

- ◉ To, calculate heights and distances, we can make use of trigonometric ratios.
- ◉ **Step 1:** Draw a **line diagram** corresponding to the problem.
- ◉ **Step 2:** Mark all known heights, distances and angles and denote unknown lengths by variables.
- ◉ **Step 3:** Use the values of various **trigonometric ratios** of the angles to obtain the unknown lengths from the known lengths.

EXTRA QUESTIONS

- Q. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° , respectively. Find the height of the poles and the distances of the point from the poles.

- Solution:**

- Let AB and CD be the poles of equal height.
- O is the point between them from where the height of elevation taken. BD is the distance between the poles.

- As per the above figure, $AB = CD$,

- $OB + OD = 80$ m

- Now,

- In right $\triangle CDO$,

- $\tan 30^\circ = CD/OD$

- $1/\sqrt{3} = CD/OD$

- $CD = OD/\sqrt{3} \dots (1)$

- In right $\triangle ABO$,

- $\tan 60^\circ = AB/OB$

- $\sqrt{3} = AB/(80-OD)$

- $AB = \sqrt{3}(80-OD)$

- $AB = CD$ (Given)

- $\sqrt{3}(80-OD) = OD/\sqrt{3}$ (Using equation (1))

- $3(80-OD) = OD$

- $240 - 3 OD = OD$

- $4 OD = 240$

- $OD = 60$

- Substituting the value of OD in equation (1)

- $CD = OD/\sqrt{3}$

- $CD = 60/\sqrt{3}$

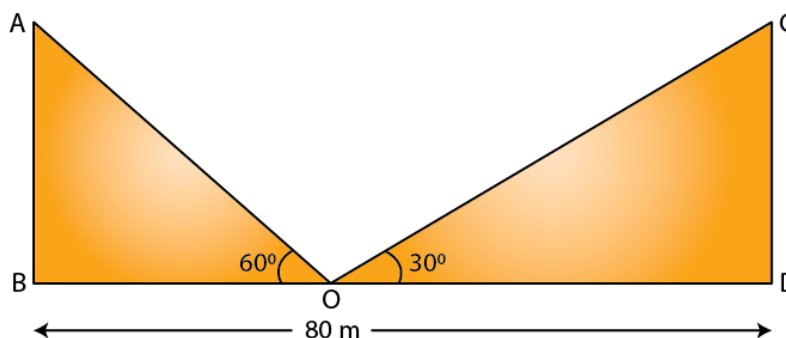
- $CD = 20\sqrt{3}$ m

- Also,

- $OB + OD = 80$ m

- $\Rightarrow OB = (80-60)$ m = 20 m

- Therefore, the height of the poles are $20\sqrt{3}$ m and distance from the point of elevation are 20 m and 60 m respectively.



HIGHLIGHTS OF CHAPTER 10

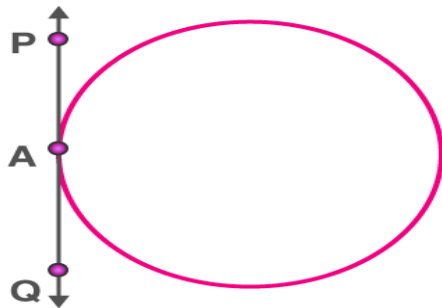
CIRCLES

- ◉ Circle and line in a plane

- ◉ For a circle and a line on a plane, there can be **three** possibilities.
- ◉ i) they can be **non-intersecting**
- ◉ ii) they can have a **single common point**: in this case, the line touches the circle.
- ◉ ii) they can have **two common points**: in this case, the line cuts the circle.

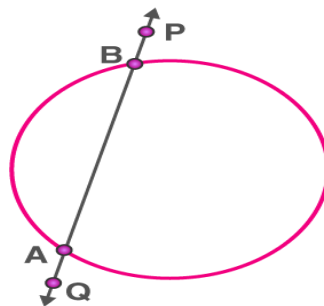
- ◉ Tangent

- ◉ A **tangent to a circle** is a line which touches the circle at exactly one point. For every point on the circle, there is a unique tangent passing



- ◉ **Secant**

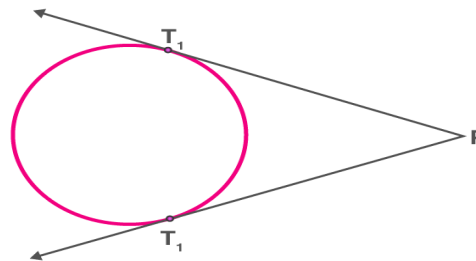
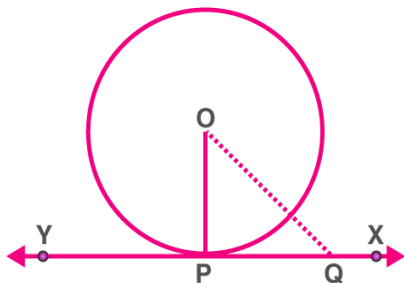
- ◉ A **secant to a circle** is a line which has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



- ◉ Two parallel tangents at most for a given secant
- ◉ For every given **secant** of a circle, there are **exactly two tangents** which are **parallel** to it and touches the circle at two **diametrically opposite points**.

- ◉ Theorems

- ◉ Tangent perpendicular to the radius at the point of contact
- ◉ Theorem: The theorem states that “the **tangent** to the circle at any point is the **perpendicular to the radius** of the circle that passes through the point of contact”.

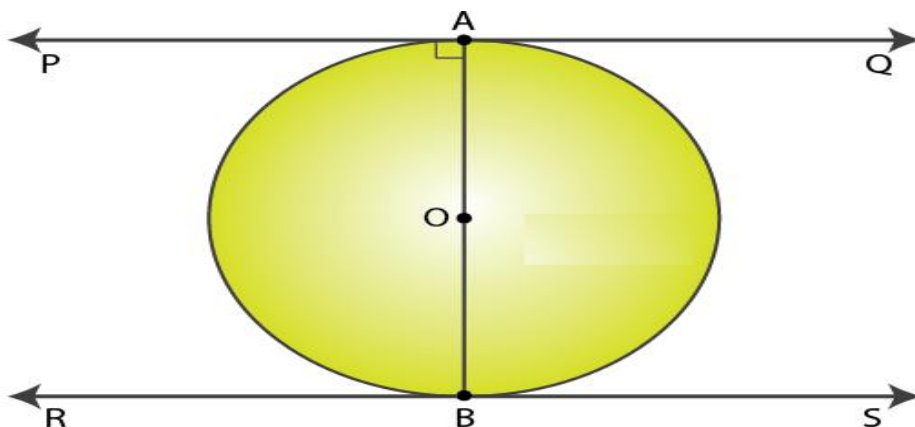


- ◉ Length of a tangent

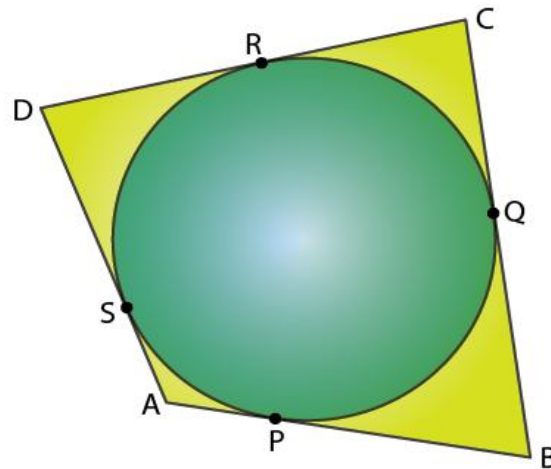
- ◉ The length of the tangent from the point (Say P) to the circle is defined as the segment of the tangent from the external point P to the point of tangency I with the circle. In this case, PI is the tangent length.
- ◉ Theorem: Two tangents are of equal length when the tangent is drawn from an external point to a circle.

EXTRA QUESTIONS

- Q. : Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
- Solution:
- First, draw a circle and connect two points A and B such that AB becomes the diameter of the circle. Now, draw two tangents PQ and RS at points A and B respectively.
- Now, both radii i.e. AO and OB are perpendicular to the tangents.
- So, OB is perpendicular to RS and OA perpendicular to PQ
- So, $\angle OAP = \angle OAQ = \angle OBR = \angle OBS = 90^\circ$
- From the above figure, angles OBR and OAQ are alternate interior angles.
- Also, $\angle OBR = \angle OAQ$ and $\angle OBS = \angle OAP$ (Since they are also alternate interior angles)
- So, it can be said that line PQ and the line RS will be parallel to each other. (Hence Proved).



- Q.: A quadrilateral ABCD is drawn to circumscribe a circle as shown in the figure. Prove that $AB + CD = AD + BC$
- Solution:**
- From this figure,
- (i) $DR = DS$
- (ii) $BP = BQ$
- (iii) $AP = AS$
- (iv) $CR = CQ$
- Since they are tangents on the circle from points D, B, A, and C respectively.
- Now, adding the LHS and RHS of the above equations we get,
- $DR + BP + AP + CR = DS + BQ + AS + CQ$
- By rearranging them we get,
- $(DR + CR) + (BP + AP) = (CQ + BQ) + (DS + AS)$
- By simplifying,
- $AD + BC = CD + AB$

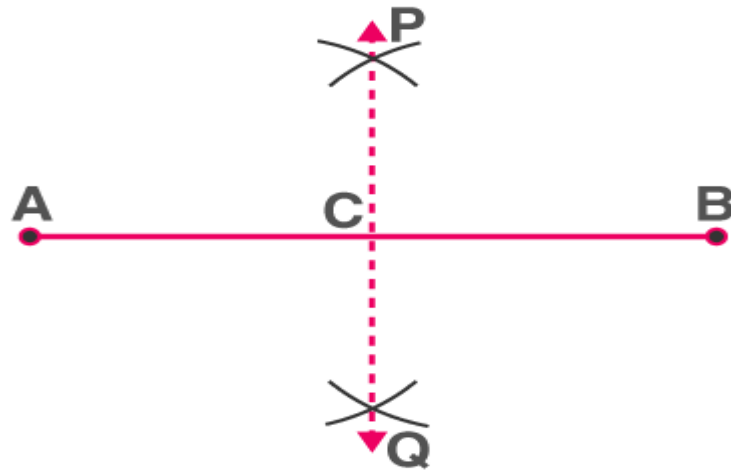


HIGHLIGHTS OF CHAPTER 11

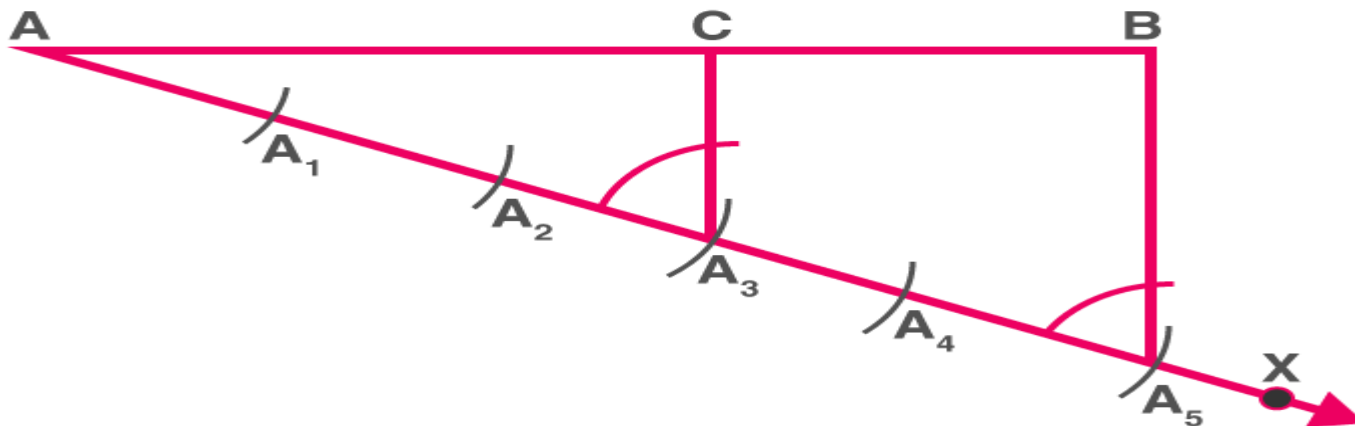
CONSTRUCTIONS

Dividing a Line Segment

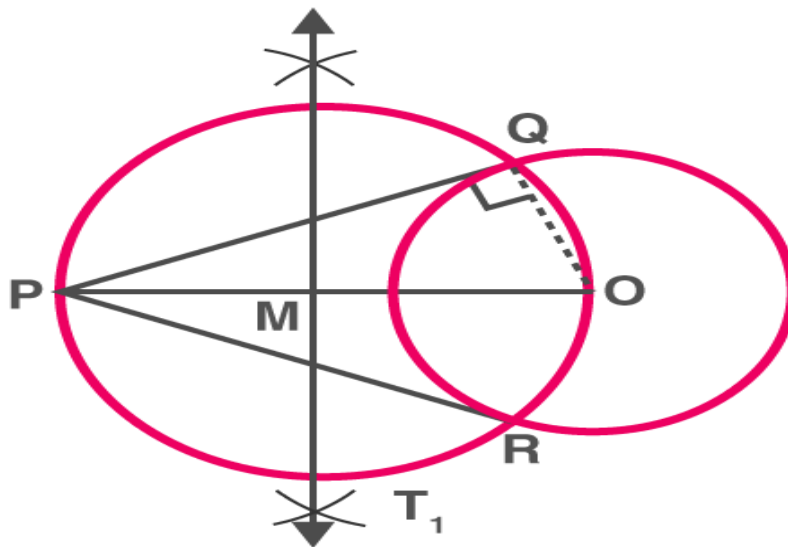
- ◉ **Bisecting a Line Segment**
- ◉ **Step 1:** With a radius of more than half the length of the line-segment, draw arcs centred at either **end** of the line segment so that they intersect on either **side** of the line segment.
- ◉ **Step 2:** Join the points of intersection. The line segment is bisected by the line segment joining the points of intersection.



- 2) Given a line segment AB, divide it in the ratio $m:n$, where both m and n are positive integers.
- Suppose we want to divide AB in the ratio 3:2 ($m=3$, $n=2$)
- Step 1:** Draw any ray AX, making an acute angle with line segment AB.
- Step 2:** Locate 5 ($= m + n$) points A_1, A_2, A_3, A_4 and A_5 on AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$
- Step 3:** Join BA_5 . ($A(m+n) = A_5$)
- Step 4:** Through the point A_3 ($m=3$), draw a line parallel to BA_5 (by making an angle equal to $\angle AA_5B$) at A_3 intersecting AB at the point C.
- Then, $AC : CB = 3 : 2$.

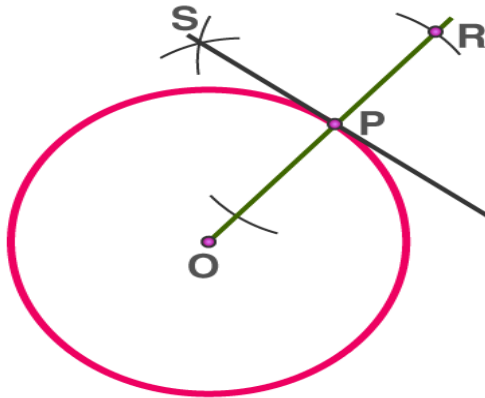


- ◉ **To construct the tangents to a circle from a point outside it.**
- ◉ Consider a circle with centre O and let P be the exterior point from which the tangents to be drawn.
- ◉ **Step 1:** Join the PO and bisect it. Let M be the midpoint of PO .
- ◉ **Step 2:** Taking M as the centre and MO (or MP) as radius, draw a circle. Let it intersect the given circle at the points Q and R .
- ◉ **Step 3:** Join PQ and PR
- ◉ **Step 3:** PQ and PR are the required tangents to the circle.

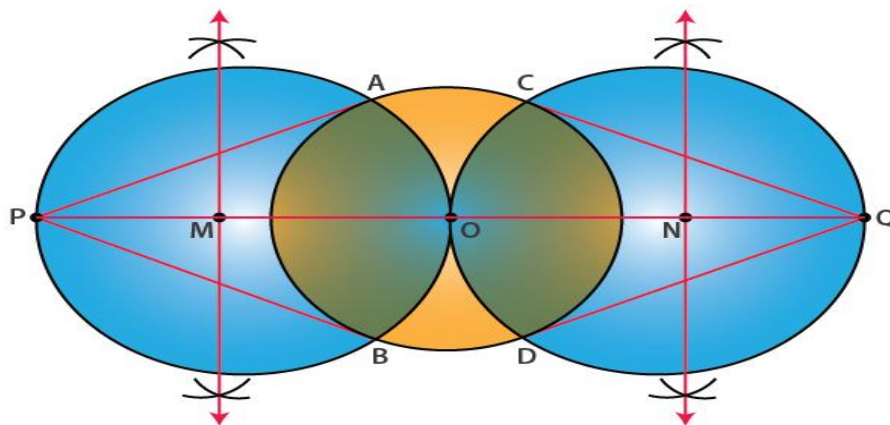


Drawing Tangents to a circle from a point on the circle

- ◉ To draw a tangent to a circle through a point on it.
- ◉ **Step 1:** Draw the radius of the circle through the required point.
- Step 2:** Draw a line perpendicular to the radius through this point. This will be tangent to the circle.



- Q.: Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.
- Solution:
- Steps of construction:
- Draw a circle with a radius of 3 cm with centre "O".
- Draw a diameter of a circle, and it extends 7 cm from the centre and mark it as P and Q.
- Draw the perpendicular bisector of the line PO and mark the midpoint as M.
- Draw a circle with M as centre and MO as the radius
- Now join the points PA and PB in which the circle with radius MO intersects the circle at points A and B.
- Now PA and PB are the required tangents.
- Similarly, from the point Q, we can draw the tangents.
- From that, QC and QD are the required tangents.



EXTRA QUESTIONS

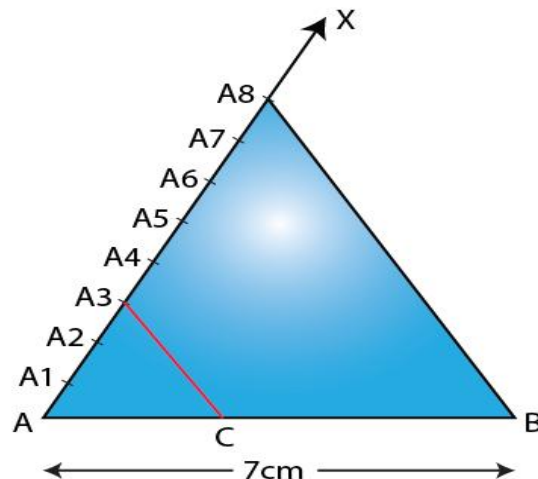
- Q.: Draw a line segment of length 7 cm. Find a point P on it which divides it in the ratio 3:5.

Solution:

Steps of construction:

1. Draw a line segment, $AB = 7$ cm.
2. Draw a ray, AX , making an acute angle downward with AB .
3. Mark the points $A_1, A_2, A_3 \dots A_8$ on AX .
4. Mark the points such that $AA_1 = A_1A_2 = A_2A_3 = \dots, A_7A_8$.
5. Join BA_8 .
6. Draw a line parallel to BA_8 through the point A_3 , to meet AB on P .

Hence $AP: PB = 3: 5$



HIGHLIGHTS OF CHAPTER 12

AREA RELATED TO CIRCLES

- ◉ **Area of a Circle**

- ◉ Area of a circle is πr^2 , where $\pi = 22/7$ or ≈ 3.14 (can be used interchangeably for problem-solving purposes) and r is the radius of the circle.

π is the ratio of the circumference of a circle to its diameter.

- ◉ **Circumference of a Circle**

- ◉ The perimeter of a circle is the distance covered by going around its boundary once. The perimeter of a circle has a special name: Circumference, which is π times the diameter which is given by the formula $2\pi r$

- ◉ **Segment of a Circle**

- ◉ A circular segment is a region of a circle which is “cut off” from the rest of the circle by a secant or a chord.

- ◉ **Sector of a Circle**

- ◉ A circle sector/ sector of a circle is defined as the region of a circle enclosed by an arc and two radii. The smaller area is called the minor sector and the larger area is called the major sector.

- ◉ **Angle of a Sector**

- ◉ The angle of a sector is that angle which is enclosed between the two radii of the sector.

- ◉ **Length of an arc of a sector**

- ◉ The length of the arc of a sector can be found by using the expression for the circumference of a circle and the angle of the sector, using the following formula:

- ◉ $L = (\theta/360^\circ) \times 2\pi r$

- ◉ Where θ is the angle of sector and r is the radius of the circle.

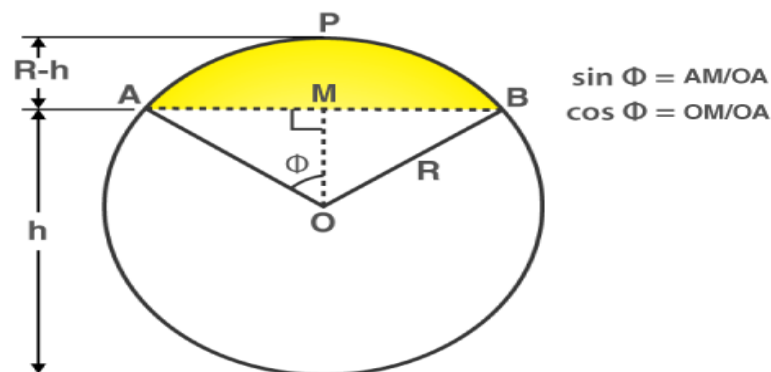
- ◉ **Area of a Sector of a Circle**

- ◉ Area of a sector is given by

- ◉ $(\theta/360^\circ) \times \pi r^2$

- ◉ where $\angle\theta$ is the angle of this sector(minor sector in the following case) and r is its radius

- **The Area of a triangle is,**
 $\text{Area} = (1/2) \times \text{base} \times \text{height}$
 If the triangle is an equilateral then
 $\text{Area} = (\sqrt{3}/4) \times a^2$ where “a” is the side length of the triangle.
- **Area of a Segment of a Circle**
- Area of segment APB (highlighted in yellow)
 $= (\text{Area of sector OAPB}) - (\text{Area of triangle AOB})$
- $= [(\theta/360^\circ) \times \pi r^2] - [(1/2) \times AB \times OM]$
- [To find the area of triangle AOB, use trigonometric ratios to find OM (height) and AB (base)] Also, Area of segment APB can be calculated directly if the angle of the sector is known using the following formula.
- $= [(\theta/360^\circ) \times \pi r^2] - [r^2 \times \sin \theta / 2 \times \cos \theta / 2]$
- Where θ is the angle of the sector and r is the radius of the circle



- ◉ **Areas of Combination of Plane figures**

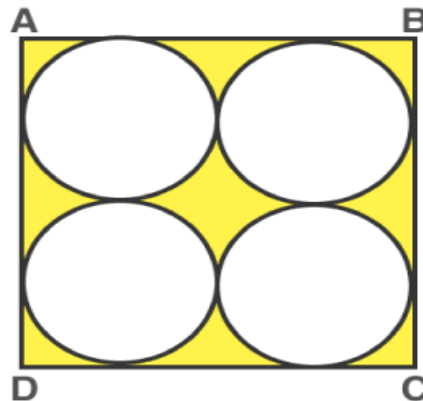
- ◉ For example: Find the area of the shaded part in the following figure: Given the ABCD is a square of side 28 cm and has four equal circles enclosed within.

- ◉ Area of the shaded region

- ◉ Looking at the figure we can visualize that the required shaded area = A(square ABCD) – 4 × A(Circle).

- ◉ Also, the diameter of each circle is 14 cm.

$$\begin{aligned} &= (l^2) - 4 \times (\pi r^2) \\ &= (28^2) - [4 \times (\pi \times 49)] \\ &= 784 - [4 \times 22/7 \times 49] \\ &= 784 - 616 \\ &= 168 \text{ cm}^2 \end{aligned}$$



EXTRA QUESTIONS

- 1. If the radius of a circle is 4.2 cm, compute its area and circumference.

Solution:

- Area of a circle = πr^2

- So, area = $\pi(4.2)^2 = 55.44 \text{ cm}^2$

- Circumference of a circle = $2\pi r$

- So, circumference = $2\pi(4.2) = 26.4 \text{ cm}$

- 2. What is the area of a circle whose circumference is 44 cm?

Solution:

- Circumference of a circle = $2\pi r$

- From the question,

- $2\pi r = 44$

- Or, $r = 22/\pi$

- Now, area of circle = $\pi r^2 = \pi \times (22/\pi)^2$

- So, area of circle = $(22 \times 22)/\pi = 154 \text{ cm}^2$

- 3. Calculate the area of a sector of angle 60° . Given, the circle is having a radius of 6 cm.

Solution:

- Given,

- The angle of the sector = 60°

- Using the formula,

- The area of sector = $(\theta/360^\circ) \times \pi r^2$

- = $(60^\circ/360^\circ) \times \pi r^2 \text{ cm}^2$

- Or, area of the sector = $6 \times 22/7 \text{ cm}^2 = 132/7 \text{ cm}^2$

- 4. A chord subtends an angle of 90° at the centre of a circle whose radius is 20 cm. Compute the area of the corresponding major segment of the circle.
- Solution:**
- Point to note:
- Area of the sector = $\frac{\theta}{360} \times \pi \times r^2$
- Base and height of the triangle formed will be = radius of the circle
- Area of the minor segment = area of the sector - area of the triangle formed
- Area of the major segment = area of the circle - area of the minor segment
- Now,
- Radius of circle = $r = 20$ cm and
- Angle subtended = $\theta = 90^\circ$
- Area of the sector = $\frac{\theta}{360} \times \pi \times r^2 = \frac{90}{360} \times \frac{22}{7} \times 20^2$
- Or, area of the sector = 314.2 cm^2
- Area of the triangle = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 20 \times 20 = 200 \text{ cm}^2$
- Area of the minor segment = $314.2 - 200 = 114.2 \text{ cm}^2$
- Area of the circle = $\pi \times r^2 = \left(\frac{22}{7}\right) \times 20^2 = 1257.14$
- Area of the major segment = $1257.14 - 114.2 = 1142.94 \text{ cm}^2$
- So, the area of the corresponding major segment of the circle = 1142.94 cm^2
- 5: A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2 . (Use $\pi = 1.7$)**

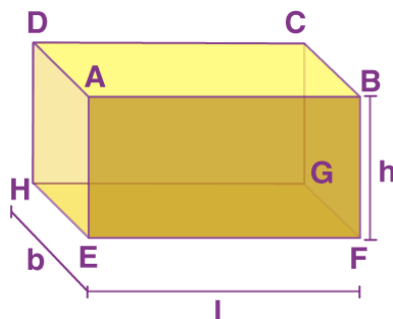
- Solution:**
- Total number of equal designs = 6
- $\angle AOB = 360^\circ / 6 = 60^\circ$
- The radius of the cover = 28 cm
- Cost of making design = Rs. 0.35 per cm^2
- Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60° , $\triangle AOB$ is an equilateral triangle. So, its area will be $\frac{\sqrt{3}}{4} \times a^2$
- Here, $a = OA$
- \therefore Area of equilateral $\triangle AOB = \frac{\sqrt{3}}{4} \times 28^2 = 333.2 \text{ cm}^2$
- Area of sector $AOB = \left(\frac{60^\circ}{360^\circ}\right) \times \pi \times r^2 \text{ cm}^2$
- = 410.66 cm^2
- Area of a single design = area of sector AOB - Area of $\triangle AOB$
- = $410.66 \text{ cm}^2 - 333.2 \text{ cm}^2 = 77.46 \text{ cm}^2$
- \therefore Area of 6 designs = $6 \times 77.46 \text{ cm}^2 = 464.76 \text{ cm}^2$
- So, the total cost of making design = $464.76 \text{ cm}^2 \times \text{Rs. } 0.35 \text{ per cm}^2$
- = Rs. 162.66

HIGHLIGHTS OF CHAPTER 13

SURFACE AREA AND VOLUME

Cuboid and its Surface Area

- The surface area of a cuboid is equal to the sum of the areas of its six rectangular faces. Consider a cuboid whose dimensions are $l \times b \times h$, respectively.
- Cuboid with length l , breadth b and height h The total surface area of the cuboid (TSA) = Sum of the areas of all its six faces
$$\text{TSA (cuboid)} = 2(l \times b) + 2(b \times h) + 2(l \times h) = 2(lb + bh + lh)$$
- Lateral surface area (LSA) is the area of all the sides apart from the top and bottom faces.
The lateral surface area of the cuboid = Area of face AEHD + Area of face BFGC + Area of face ABFE + Area of face DHGC
$$\text{LSA (cuboid)} = 2(b \times h) + 2(l \times h) = 2h(l + b)$$
- Length of diagonal of a cuboid $= \sqrt{l^2 + b^2 + h^2}$



- ◉ **Cube and its Surface Area**

- ◉ For a cube, length = breadth = height

- ◉ Cube with length l , **TSA (cube) = $2 \times (3l^2) = 6l^2$**

- ◉ **Lateral surface area of cube = $2(l \times l + l \times l) = 4l^2$**

Note: Diagonal of a cube = $\sqrt{3}l$

- ◉ **Cylinder and its Surface Area**

- ◉ Take a cylinder of base radius r and height h units. The curved surface of this cylinder, if opened along the diameter ($d = 2r$) of the circular base can be transformed into a rectangle of length $2\pi r$ and height h units. Thus,

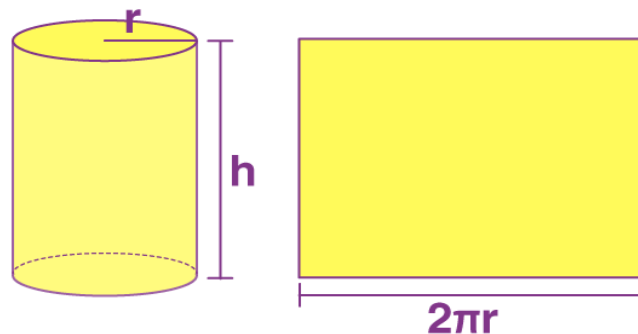
- ◉ Transformation of a Cylinder into a rectangle.

- ◉ **CSA of a cylinder = $2\pi \times r \times h$**

TSA of a cylinder = $2\pi \times r \times h$ + area of two circular bases

= $2\pi \times r \times h + 2\pi r^2$

TSA of a cylinder = $2\pi r(h + r)$



- ◉ **Right Circular Cone and its Surface Area**

- ◉ Consider a right circular cone with slant length l , radius r and height h .

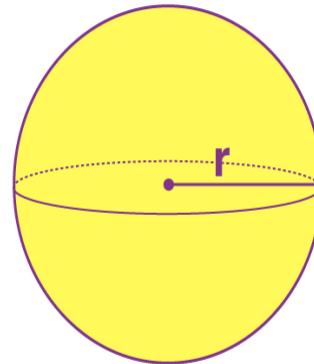
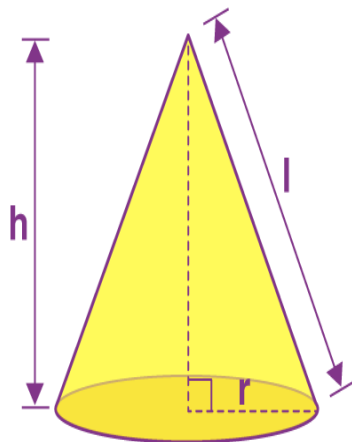
- ◉ **CSA of right circular cone = πrl**

$$\text{TSA} = \text{CSA} + \text{area of base} = \pi rl + \pi r^2 = \pi r(l + r)$$

- ◉ **Sphere and its Surface Area**

- ◉ For a sphere of radius r

- ◉ **Curved Surface Area (CSA) = Total Surface Area (TSA) = $4\pi r^2$**



Volume of a cuboid = (base area) \times height = $(lb)h = lbh$

- ◉ **Volume of a cube** = base area \times height

Since all dimensions of a cube are identical, volume = l^3

Where l is the length of the edge of the cube.

- ◉ **Volume of a cylinder** = Base area \times height = $(\pi r^2) \times h = \pi r^2 h$

- ◉ The volume of a Right circular cone is $1/3$ times that of a cylinder of same height and base.

The volume of a Right circular cone = $(1/3)\pi r^2 h$

Where r is the radius of the base and h is the height of the cone.

- ◉ **The volume of a sphere of radius r** = $(4/3)\pi r^3$

- ◉ A hemisphere is half of a sphere.

\therefore **CSA of a hemisphere of radius r** = $2\pi r^2$

Total Surface Area = curved surface area + area of the base circle

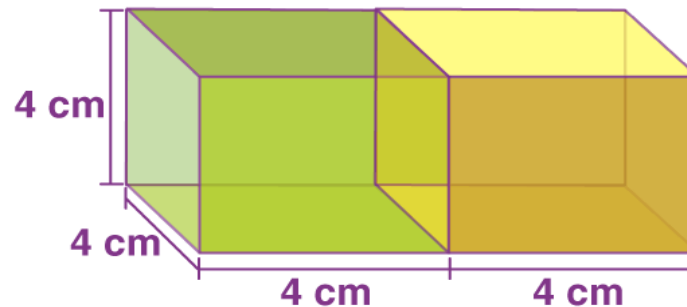
\Rightarrow **TSA** = $3\pi r^2$

- ◉ The volume (V) of a hemisphere will be half of that of a sphere.

\therefore **The volume of the hemisphere of radius r** = $(2/3)\pi r^3$

- ◉ **Surface Area of Combined Figures**

- ◉ Areas of complex figures can be broken down and analysed as simpler known shapes. By finding the areas of these known shapes, we can find out the required area of the unknown figure. Example: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.
Length of each cube = $\sqrt[3]{64} = 4 \text{ cm}$
Since these cubes are joined adjacently, they form a cuboid whose length $l = 8 \text{ cm}$. But height and breadth will remain same = 4 cm .
- ◉ Combination of 2 equal cubes.: The new surface area, $\text{TSA} = 2(lb + bh + lh)$
- ◉ $\text{TSA} = 2(8 \times 4 + 4 \times 4 + 8 \times 4)$
- ◉ $= 2(32 + 16 + 32)$
- ◉ $= 2(80)$
- ◉ $\text{TSA} = 160 \text{ cm}^2$



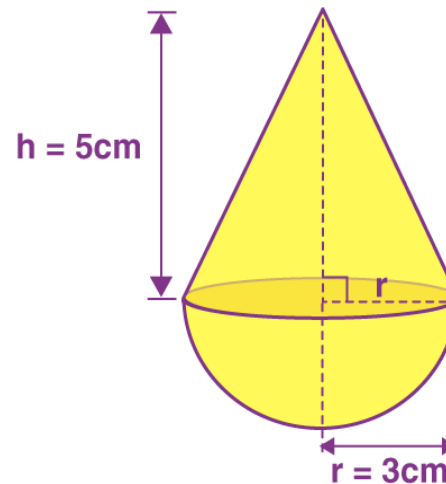
- ◉ **Volume of Combined Solids**

- ◉ The volume of complex objects can be simplified by visualising it as a combination of shapes of known solids.

Example: A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the height of the cone is equal to 5 cm.

This can be visualised as follows :

- ◉ Volume of combined solids $V(\text{solid}) = V(\text{Cone}) + V(\text{hemisphere})$
- ◉ $V(\text{solid}) = (1/3)\pi r^2 h + (2/3)\pi r^3$
- ◉ $V(\text{solid}) = (1/3)\pi(9)(5) + (2/3)\pi(27)$
- ◉ $V(\text{solid}) = 33\pi \text{ cm}^3$



IMPORTANT QUESTIONS

- Q. : Selvi's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (an underground tank) which is in the shape of a cuboid. The sump has dimensions $1.57 \text{ m} \times 1.44 \text{ m} \times 95 \text{ cm}$. The overhead tank has its radius 60 cm and height 95 cm . Find the height of the water left in the sump after the overhead tank has been completely filled with water from the sump which had been full. Compare the capacity of the tank with that of the sump. (Use $\pi = 3.14$)
- Solution:
- The volume of water in the overhead tank equals the volume of the water removed from the sump.
- Now, the volume of water in the overhead tank (cylinder) $= \pi r^2 h$
- $= 3.14 \times 0.6 \times 0.6 \times 0.95 \text{ m}^3$
- The volume of water in the sump when full $= l \times b \times h = 1.57 \times 1.44 \times 0.95 \text{ m}^3$
- The volume of water left in the sump after filling the tank
- $= [(1.57 \times 1.44 \times 0.95) - (3.14 \times 0.6 \times 0.6 \times 0.95)] \text{ m}^3 = (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2) \text{ m}^3$
- Height of the water left in the sump $= (\text{volume of water left in the sump}) / (l \times b)$
- $= (1.57 \times 0.6 \times 0.6 \times 0.95 \times 2) / (1.57 \times 1.44)$
- $= 0.475 \text{ m}$
- $= 47.5 \text{ cm}$
- Capacity of tank / Capacity of sump $= (3.14 \times 0.6 \times 0.6 \times 0.95) / (1.57 \times 1.44 \times 0.95)$
- $= 1/2$
- Therefore, the capacity of the tank is half the capacity of the sump.

Q.: Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.

Solution:

For Sphere 1:

Radius (r_1) = 6 cm

\therefore Volume (V_1) = $(4/3) \times \pi \times r_1^3$

For Sphere 2:

Radius (r_2) = 8 cm

\therefore Volume (V_2) = $(4/3) \times \pi \times r_2^3$

For Sphere 3:

Radius (r_3) = 10 cm

\therefore Volume (V_3) = $(4/3) \times \pi \times r_3^3$

Also, let the radius of the resulting sphere be “r”

Now, Volume of resulting sphere = $V_1 + V_2 + V_3$

$(4/3) \times \pi \times r^3 = (4/3) \times \pi \times r_1^3 + (4/3) \times \pi \times r_2^3 + (4/3) \times \pi \times r_3^3$

$r^3 = 6^3 + 8^3 + 10^3$

$r^3 = 1728$

$r = 12$ cm

HIGHLIGHTS OF CHAPTER 14

STATISTICS

- ◉ **Ungrouped Data**

- ◉ Ungrouped data is data in its original or raw form. The observations are not classified into groups.
- ◉ For example, the ages of everyone present in a classroom of kindergarten kids with the teacher is as follows:
- ◉ 3, 3, 4, 3, 5, 4, 3, 3, 4, 3, 3, 3, 4, 3, 27.
- ◉ This data shows that there is one adult present in this class and that is the teacher. Ungrouped data is easy to work with when the data set is small.

- ◉ **Grouped Data**

- ◉ In grouped data, observations are organized in groups.
- ◉ For example, a class of students got different marks in a school exam. The data is tabulated as follows:

Mark interval	0-20	21-40	41-60	61-80	81-100
No. of Students	13	9	36	32	10

- ◉ This shows how many students got the particular mark range. Grouped data is easier to work with when a large amount of data is present.

- ◉ **Frequency** is the number of times a particular observation occurs in data.
- ◉ **Class Interval**
- ◉ Data can be grouped into class intervals such that all observations in that range belong to that class.
- ◉ **Class width** = upper class limit - lower class limit
- ◉ **Mean**
- ◉ Finding the mean for Grouped Data when class Intervals are not given
- ◉ For grouped data without class intervals,
- ◉ $\text{Mean} = \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$ where f_i is the frequency of i th observation x_i .
- ◉ Finding the mean for Grouped Data when class Intervals are given
- ◉ For grouped data with class intervals,
- ◉ $\text{Mean} = \bar{x} = \frac{\sum x_i f_i}{\sum f_i}$
- ◉ Where f_i is the frequency of i th class whose class mark is x_i .
 $\text{Classmark} = (\text{Upper Class Limit} + \text{Lower Class Limit}) / 2$

- ⦿ **Direct method of finding mean**
- ⦿ Step 1: Classify the data into intervals and find the corresponding frequency of each class.
- ⦿ Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.
- ⦿ Step 3: Tabulate the product of class mark and its corresponding frequency for each class. Calculate their sum ($\sum x_i f_i$).
- ⦿ Step 4: Divide the above sum by the sum of frequencies ($\sum f_i$) to get the mean.

- ⦿ **Assumed mean method of finding mean**
- ⦿ Step 1: Classify the data into intervals and find the corresponding frequency of each class.
- ⦿ Step 2: Find the class mark by taking the midpoint of the upper and lower class limits.
- ⦿ Step 3: Take one of the x_i 's (usually one in the middle) as assumed mean and denote it by 'a'.
- ⦿ Step 4: Find the deviation of 'a' from each of the x_i 's
 $d_i = x_i - a$
- ⦿ Step 5: Find the mean of the deviations
- ⦿ $d^- = \sum x_i d_i / \sum f_i$
- ⦿ Step 6: Calculate the mean as
 $\bar{x} = a + \sum x_i d_i / \sum f_i$
- ⦿

- ◉ Cumulative frequency is obtained by adding all the frequencies up to a certain point.
- ◉ Finding median for Grouped Data when class Intervals are given
- ◉ Step 1: find the cumulative frequency for all class intervals.
Step 2: the median class is the class whose cumulative frequency is greater than or nearest to $n/2$, where n is the number of observations.
Step 3: Median = $l + [(n/2 - cf) / f] \times h$
- ◉ Where,
l = lower limit of median class,
n = number of observations,
cf = cumulative frequency of class preceding the median class,
f = frequency of median class,
h = class size (assuming class size to be equal).

◉ Mode

◉ Finding mode for Grouped Data when class intervals are not given

- ◉ In grouped data without class intervals, the observation having the largest frequency is the mode.

◉ Finding mode for Ungrouped Data

- ◉ For ungrouped data, the mode can be found out by counting the observations and using tally marks to construct a frequency table.

The observation having the largest frequency is the **mode**.

◉ Finding mode for Grouped Data when class intervals are given

- ◉ For, grouped data, the class having the highest frequency is called the modal class. The mode can be calculated using the following formula. The formula is valid for equal class intervals and when the modal class is unique.

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$$

- ◉ Where,

l = lower limit of modal class

h = class width

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class.

- Mean, mode and median are connected by the empirical relationship
- $3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$

HIGHLIGHTS OF CHAPTER 15

PROBABILITY

- ◉ **Event and outcome**
- ◉ An **Outcome** is a result of a random experiment. For example, when we roll a dice getting six is an outcome.
An **Event** is a set of outcomes. For example when we roll dice the probability of getting a number less than five is an event.
Note: An Event can have a single outcome.
- ◉ Experimental probability can be applied to any event associated with an experiment that is repeated a large number of times.
A trial is when the experiment is performed once. It is also known as **empirical probability**.

Experimental or empirical probability: $P(E) = \frac{\text{Number of trials where the event occurred}}{\text{Total Number of Trials}}$

- ◉ **Theoretical Probability, $P(E) = \frac{\text{Number of Outcomes Favourable to E}}{\text{Number of all possible outcomes of the experiment}}$**
- ◉ Here we assume that the outcomes of the experiment are **equally likely**.

- ◉ **Elementary Event**

- ◉ An event having only **one outcome** of the experiment is called an **elementary event**.

Example: Take the experiment of tossing a coin n number of times. One trial of this experiment has two possible outcomes: Heads(H) or Tails(T). So for an individual toss, it has only one outcome, i.e Heads or Tails.

- ◉ **Sum of Probabilities**

- ◉ The **sum** of the probabilities of all the **elementary events** of an experiment is **one**.

Example: take the coin-tossing experiment. $P(\text{Heads}) + P(\text{Tails})$

- ◉ $= (1/2) + (1/2) = 1$

- ◉ **Impossible event**

- ◉ An event that has **no chance of occurring** is called an **Impossible event**, i.e. $P(E) = 0$.

E.g: Probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.

- ◉ **Sure event**

- ◉ An event that has a **100% probability** of occurrence is called a **sure event**. The probability of occurrence of a **sure event** is **one**.

E.g: What is the probability that a number obtained after throwing a die is less than 7?

So, $P(E) = P(\text{Getting a number less than 7}) = 6/6 = 1$

- ◉ Range of Probability of an event
- ◉ The range of probability of an event lies between 0 and 1 inclusive of 0 and 1, i.e. $0 \leq P(E) \leq 1$.
- ◉ Geometric Probability
- ◉ Geometric probability is the calculation of the likelihood that one will hit a particular area of a figure. It is calculated by dividing the desired area by the total area. In the case of Geometrical probability, there are infinite outcomes.
- ◉ Complementary Events
- ◉ Complementary events are two outcomes of an event that are the only two possible outcomes. This is like flipping a coin and getting heads or tails. $P(E) + P(E^-) = 1$, where E and E^- are complementary events. The event E^- , representing 'not E ', is called the **complement** of the event E .

IMPORTANT QUESTIONS

- Q. 1: Two dice are thrown at the same time. Find the probability of getting
- (i) the same number on both dice.
- (ii) different numbers on both dice.
- Solution:**
- Given that, Two dice are thrown at the same time.
- So, the total number of possible outcomes $n(S) = 6^2 = 36$
- (i) Getting the same number on both dice:
- Let A be the event of getting the same number on both dice.
- Possible outcomes are (1,1), (2,2), (3, 3), (4, 4), (5, 5) and (6, 6).
- Number of possible outcomes = $n(A) = 6$
- Hence, the required probability $= P(A) = n(A)/n(S)$
- $= 6/36$
- $= 1/6$
- (ii) Getting a different number on both dice.
- Let B be the event of getting a different number on both dice.
- Number of possible outcomes $n(B) = 36$ - Number of possible outcomes for the same number on both dice
- $= 36 - 6 = 30$
- Hence, the required probability $= P(B) = n(B)/n(S)$
- $= 30/36$
- $= 5/6$

- Q.: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will

- (i) be an ace,
- (ii) not be an ace.

- Solution:**

- Well-shuffling ensures equally likely outcomes.

- (i) Card drawn is an ace

- There are 4 aces in a deck.

- Let E be the event 'the card is an ace'.

- The number of outcomes favourable to E = $n(E) = 4$

- The number of possible outcomes = Total number of cards = $n(S) = 52$

- Therefore, $P(E) = n(E)/n(S) = 4/52 = 1/13$

- (ii) Card drawn is not an ace

- Let F be the event 'card drawn is not an ace'.

- The number of outcomes favourable to the event F = $n(F) = 52 - 4 = 48$

- Therefore, $P(F) = n(F)/n(S) = 48/52 = 12/13$

- Q. An integer is chosen between 0 and 100. What is the probability that it is**

- (i) divisible by 7?

- (ii) not divisible by 7?

- Solution:**

- Number of integers between 0 and 100 = $n(S) = 99$

- (i) Let E be the event 'integer divisible by 7'

- Favourable outcomes to the event E = 7, 14, 21, ..., 98

- Number of favourable outcomes = $n(E) = 14$

- Probability = $P(E) = n(E)/n(S) = 14/99$

- (ii) Let F be the event 'integer not divisible by 7'

- Number of favourable outcomes to the event F = 99 - Number of integers divisible by 7

- = $99 - 14 = 85$

- Hence, the required probability = $P(F) = n(F)/n(S) = 85/99$