

St. Mary's School, Dwarka
Holiday Homework
Class XI
Subject: Mathematics

Worksheet 1 (Chapter-Sets)

Instructions:

Question 1-34 carry 1 mark each – Choose the correct option:

1. If $A = \{x: x = 3n + 1, 2 \leq n \leq 5\}$, then number of subsets of A is:
(a) 4 (b) 16 (c) 8 (d) none of these
2. Which of the following is a null set?
(a) $\{x: |x| = 4, x \in \mathbb{N}\}$ (b) $\{x: x^2 + 2x + 1 = 0, x \in \mathbb{R}\}$
(c) $\{x: |x| < 1, x \in \mathbb{N}\}$ (d) none of these
3. If $a\mathbb{N} = \{ax: x \in \mathbb{N}\}$, then $(3\mathbb{N} \cap 7\mathbb{N}) =$
(a) $3\mathbb{N}$ (b) $7\mathbb{N}$ (c) \mathbb{N} (d) $21\mathbb{N}$
4. If X and Y are two sets, then $X \cap (X \cap Y)$ equals
(a) X (b) Y (c) \emptyset (d) none of these
5. For two sets A and B, $A \cap (A \cup B)$ is equal to
(a) A (b) B (c) \emptyset (d) none of these
6. Let $A = \{1, 2, 3\}$, $B = \{3, 4\}$, $C = \{4, 5, 6\}$, then $A \cup (B \cap C)$ is
(a) $\{3\}$ (b) $\{1, 2, 3, 4\}$
(c) $\{1, 2, 5, 6\}$ (d) $\{1, 2, 3, 4, 5, 6\}$
7. If A, B, C be three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then
(a) $A = B$ (b) $B = C$ (c) $A = C$ (d) $A = B = C$
8. If a set A has n elements, then the total number of subsets of A is
(a) n (b) n^2 (c) 2^n (d) 2n
9. Two finite sets have m and n elements and total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n are
(a) 7, 6 (b) 6, 3 (c) 5, 1 (d) 8, 7
10. A set contains n elements. Then its power set contains
(a) n elements (b) 2^{n-1} elements
(c) n^2 elements (d) none of these
11. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then the total number of students who have passed in Physics only is

- (a) 33 (b) 22 (c) 45 (d) 10

12. Which of the following statements is true?

- (a) $\{3, 5\} \in \{1, 3, 5\}$ (b) $\{3\} \in \{1, 3, 5\}$ (c) $3 \in \{1, 3, 5\}$ (d) $3 \subset \{1, 3, 5\}$

13. If $A = \{1, 2, 3, 4, 5\}$, $B = \{x: x^2 - 5x + 6 = 0\}$, $C = \{x: x^2 = 4 \text{ or } x = 1\}$, then

- (a) $A \cap (B \cap C) = \phi$ (b) $A \cap (B \cap C) = \phi$
 (c) both (a) and (b) (d) none of these

14. Let $S = \{0, 1, 5, 4, 7\}$. Then total number of subsets of S is

- (a) 61 (b) 20 (c) 40 (d) 32

15. The set of real numbers is

- (a) finite
 (b) is a subset of the set of complex numbers
 (c) is a subset of the set of natural numbers
 (d) none of these

16. What is singleton?

- (a) It is a single set
 (b) It is a singular matrix
 (c) It is a set with a single element
 (d) It is a null set

17. The set $A = \{x: x \in \mathbb{R}, x^2 = 16 \text{ and } 2x = 6\}$ equals

- (a) $\langle I \rangle$ (b) $\{14, 3, 4\}$ (c) $\{3\}$ (d) $\{4\}$

18. Set A and B have 3 and 6 elements, respectively. What can be the minimum number of elements in $A \cup B$?

- (a) 18 (b) 9 (c) 6 (d) 3

19. If $A = \{1, 2, 3, 4, 5\}$, then the number of proper subsets of A is

- (a) 120 (b) 30 (c) 31 (d) 32

20. Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, $C = \{a, b, d, e\}$, then $n(B \cup C)$

- (a) $\{c\}$ (b) $\{a, b, c\}$ (c) $\{b, c, d\}$ (d) $\{a, b, d, e\}$

21. The number of non-empty subsets of the set $\{1, 2, 3, 4\}$ is

- (a) 15 (b) 14 (c) 16 (d) 17

22. For any non-negative integer n if a set A has n elements, then the set of all subsets of A has

- (a) $2n$ elements (b) n elements
 (c) 2^n elements (d) 2^{n+1} elements

23. If A , B and C are non-empty subsets of a set, then $(A - B) \cup (B - A)$ equals

- (a) $(A \cap B) \cup (A \cup B)$
 (b) $(A \cap B) - (A \cap B)$
 (c) $A - (A \cap B)$
 (d) $(A \cup B) - B$

24. $A = \{x: x \neq x\}$ represents
 (a) $\{x\}$ (b) $\{I\}$ (c) $\{\cdot\}$ (d) $\{0\}$
25. If a set A has n elements, then the number of elements in the power set of A's is
 (a) 2211 (b) $2^{11} + 1$ (c) n^2 (d) 211
26. The number of elements in the power set of $\{1, 2, 3, 4, 5\}$ are
 (a) 16 (b) 32 (c) 64 (d) none of these
27. In a group of 520 persons each one speaks at least one of the two languages Hindi and English. 360 can speak Hindi only and 130 can speak English only. The number of persons who can speak both Hindi and English is
 (a) 20 (b) 25 (c) 30 (d) 35
28. If A, B, C are 3 non-empty sets, then $(A \cap B) \cap (B \cap C) \cap (C \cap A)$ is equal to
 (a) $A \cap B \cap C$ (b) $A \cup B \cup C$ (c) ϕ (d) none of these
29. If A is the set of the divisors of the number 15, B is the set of prime numbers smaller than 10 and C is the set of even numbers smaller than 9, then $(A \cup C) \cap B$ is the following set
 (a) $\{1, 3, 5\}$ (b) $\{1, 2, 3\}$ (c) $\{2, 3, 5\}$ (d) $\{2, 5\}$
30. There are 600 students in a school. If 400 of them can speak Telugu, 300 can speak Hindi, then the number of students who can speak both Telugu and Hindi are
 (a) 100 (b) 200 (c) 300 (d) 400
31. Out of 40 children, 30 can swim, 27 can play chess and only 5 can do neither. How many can swim only?
 (a) 30 (b) 22 (c) 12 (d) 8
32. In a class containing 120 students 65 students drink tea and 84 students drink coffee. If x students drink both tea and coffee, what is the value of x?
 (a) 36 (b) 65 (c) $29 < x < 65$ (d) $29 < x < 84$
33. $(A \cup B) \cap (A' \cap B')$ is equal to
 (a) ϕ (b) n (c) A (d) B
34. $(A \cup B) \cap (A \cap B')$ is equal to
 (a) B (b) A (c) $A \cup B$ (d) ϕ

Worksheet 2 (Chapter-Sets)

Instructions:

Question 1-13 carry 2 mark each.

Question 13-24 carry 4 mark each.

1. If A and B are two sets such that $n(A \cup B) = 50$, $n(A) = 28$ and $n(B) = 32$, find $n(A \cap B)$.
2. If A and B are two sets such that $A \cup B$ has 18 elements, A has 8 elements and B has 15 elements, how many elements does $A \cap B$ have?
3. In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach physics and mathematics. How many teach physics?
4. If S and T are two sets such that S has 21 elements, T has 32 elements and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?
5. In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi? How many people speak English?
6. In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many can speak both Hindi and English?
7. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?
8. In a committee 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?
9. In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?
10. In a group of people, 50 speak both English and Hindi and 30 people speak English but not Hindi. All the people speak at least one of the two languages. How many people speak English?
11. If 45% of the students of a class have offered Mathematics and 85% of them Biology, find the percentage of students who offered Biology only?
12. In a group, every member knows at least one of the languages-Hindi and Urdu. 100 members know Hindi, 50 know Urdu and 25 of them both Hindi and Urdu. How many members are there in the group?

13. In a survey of 600 students in a school, 150 students were found to be drinking Tea and 225 drinking Coffee, 100 were drinking both Tea and Coffee. Find how many students were drinking neither Tea nor Coffee?

14. In a class of 35 students, 17 have taken Mathematics, 10 have taken mathematics but not physics. Find the number of students who have taken both mathematics and physics and the number of students who have taken physics but not mathematics. It is given that each student has taken either mathematics or physics or both.

15. Out of 500 car owners investigated, 400 owned car A and 200 owned car B. 50 owned both cars A and B. Is this data correct?

16. In a group of 45 students, 22 can speak Hindi only, 12 can speak English only. How many can speak both Hindi and English?

17. In a town with population 5000, 2800 read the Times of India, 2300 read Indian Express and 400 read both. How many read neither?

18. An investigator interviewed 100 students to determine their preferences for the three drinks: Milk (M), Coffee (C) and Tea (T). He reported the following: 10 students had none of the three drinks M, C, T, 20 had M and C, 30 had C and T, 25 had M and T; 12 had M only; 5 had C only; 8 had T only. Using a Venn Diagram, find how many did not take any of the three drinks?

19. In a survey of 25 students, it was found that 15 had taken mathematics, 12 had taken physics and 11 had taken chemistry, 5 had taken mathematics and chemistry, 9 had taken mathematics and physics, 4 had taken physics and chemistry and 3 had taken all the three subjects. Find the number of students who had taken:

(i) only chemistry

(ii) only Mathematics

(iii) only physics

(iv) physics and chemistry but not mathematics

(v) mathematics and physics but not chemistry

(vi) only one of the subjects

(vii) at least one of the three subjects

(viii) none of three subjects.

20. In a survey of 100 persons it was found that 28 read magazine A, 30 read magazine B, 42 read magazine C, 8 read magazines A and B, 10 read magazines A and C, 5 read magazines B and C and 3 read all the three magazines. Find

(i) How many read none of the three magazines?

(ii) How many read magazine C only?

21. In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B; 2 people liked products C and A; 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.

22. In a survey of 60 people, it was found that 25 people read Newspaper H, 26 read Newspaper T, 26 read Newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all the three newspapers. Find

(i) the number of people who read at least one of the newspapers

(ii) the number of people who read exactly one newspaper.

23. In a survey of 100 students, the number of students studying the various

languages were found to be English only 18, English but not Hindi 23, English and Sanskrit 8, English 26, Sanskrit 48, Sanskrit and Hindi 8, no language 24. Find:

(i) How many students were studying Hindi?

(ii) How many students were studying English and Hindi?

24. In a group of children, 35 play football out of which 20 play football only, 22 play hockey, 25 play crickets out of which 11 play crickets only. Out of these 7 play cricket and football but not hockey, 3 play football and hockey but not cricket and 12 play football and cricket both. How many play all the three games? How many play cricket and hockey but not football, how many play hockey only? What is the total number of children in the group? [HOTS]

Worksheet 3 (Linear Equations)

Instructions:

Question 1-8 carry 2 mark each.

Question 1-8 (HOTS) carry 4 mark each.

1. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

2. A company manufactures cassettes and its cost and revenue functions are $C(x) = 26000 + 30x$ and $R(x) = 43x$ respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realize some profit.

3. Solve $\frac{x-2}{x+5} > 2$

4. Solve $\frac{2x+3}{4} - 3 < \frac{x-4}{3} - 2, x \in R$

5. Solve $-5 \leq \frac{2-3x}{4} \leq 9, x \in R$

6. Solve the following system of inequations

$$|x-1| \leq 1, |x| \geq 1$$

7. Solve the following system of inequalities

$$\begin{aligned} 5x+4y &\leq 40 \\ x &\geq 2 \\ y &\geq 3 \end{aligned}$$

Graphically:

8. Solve the following system of inequalities

$$\begin{aligned} x+2y &\leq 8 \\ 2x+y &\leq 8 \\ x &\geq 0 \\ y &\geq 0 \end{aligned}$$

Graphically:

HOTS

1. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres 3% solution will have to be added?

2. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

3. Solve $|2x - 3| \leq 11, |x - 2| \geq 3$

4. Solve $\frac{|x| - 1}{|x| - 2} \in [0, 1] \cap \mathbb{R}, x \neq \pm 2$

5. Solve for x : $|x| + |x - 1| > 3$

Solve the following system of inequalities graphically:

6. $2x + y \leq 24, x + y < 11, 2x + 5y \leq 40, x \geq 0, y \geq 0$

7. $3x + 2y \geq 24, 3x + y \leq 15, x \geq 4$

8. $x - 2y \leq 3, 3x + 4y > 12, x \geq 0, y \geq 1$

Worksheet 4 (Mathematical Induction)

Important Points and Definitions:

1. Principle of Mathematical Induction:

A statement $P(n)$ is true for all $n \in \mathbb{N}$, where \mathbb{N} denotes the set of all-natural numbers, provided:

(a) $P(1)$ is true and

(b) If $P(k)$ is true then $P(k + 1)$ is true.

2. If any one of the above conditions fails to hold, then statement is not true for all natural numbers.

Questions 1-6 carry 4 mark each.

Q.1 If $P(n)$ is the statement “ $2^{3n} - 1$ is an integral multiple of 7”, prove that $P(1)$, $P(2)$ and $P(3)$ are true.

Q.2 Prove the following by the principle of mathematical induction for every natural number n.

(a) $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

(b) $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

(c) $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

Q.3 Show that $1.2 + 2.3 + 3.4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Q.4 Show that $2^{3n} - 1$ is divisible by 7.

Q.5 Prove the following by the principle of mathematical induction for every natural number n.

(a) $3^{2n} - 1$ is divisible by 8.

(b) $10^{2n-1} + 1$ is divisible by 11

(c) $1 + 2 + 3 + \dots + n < \frac{1}{8}(2n+1)^2$.

(d) $x^n - y^n$ is divisible by $(x - y)$, where $x - y \neq 0$.

Q.6 By the principle of mathematical induction, prove that the number of all subsets of a set n elements is 2^n , where n is a natural number

Q.7 prove that $n(n+1)(n+2)$ is divisible by 2 by using principle of Mathematical Induction.