St. Mary's School, Dwarka

Holiday Homework

Class -XII

Subject: Mathematics

WORKSHEET-1A

CHAPTER-2 INVERSE TRIGONOMETRIC FUNCTIONS

Instructions:

Question 1-22 carry 1 mark each.

Mark the correct alternative in each of the following:

Q.1 If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x =

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{2}$ (d) none of these

Q.2 $\sin\left[\cot^{-1}\left\{\tan\left(\cos^{-1} x\right)\right\}\right]$ is equal to

(a) x (b) $\sqrt{1-x^2}$ (c) $\frac{1}{x}$ (d) none of these

Q.3 The number of solutions of the equation

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

(a) 2 (b) 3 (c) 1 (d) none of these

Q.4 If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then

(a) 4a = 3b (b) 3a = 4b (c) $\alpha - \beta = \frac{7\pi}{12}$ (d) none of these

Q.5 If x < 0, y < 0 such that xy = 1, then $\tan^{-1}x + \tan^{-1}y$ equals

(a) $\frac{\pi}{2}$ (b) - $\frac{\pi}{2}$ (c) -p (d) none of these

Q.6 If $u = \cot^{-1}\sqrt{\tan\theta} - \tan^{-1}\sqrt{\tan\theta}$ then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) =$

(a) $\sqrt{\tan\theta}$ (b) $\sqrt{\cot\theta}$ (c) tanq (d) cot q

Q.7 If $\alpha = \tan^{-1}\left(\frac{\sqrt{3}x}{2y - x}\right), \beta = \tan^{-1}\left(\frac{2x - y}{\sqrt{3}y}\right)$, then a - b =

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{3}$

Q.8 $\tan^{-1}\frac{1}{11} + \tan^{-1}\frac{2}{11}$ is equal to

(a) 0 (b) 1/2 (c) -1 (d) none of these

Q.9 If $\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = \theta$, then $9x^2 - 12xy \cos q + 4y^2$ is equal to

(a) 36 (b) -36 $\sin^2$ q (c) 36 $\sin^2$ q (d) 36 $\cos^2$ q

Q.10 If $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$, then x =

(a) 5 (b) 1/5 (c) 5/14 (d) 14/5

Q.11 The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is

(a) $\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{\pi}{10}$ (d) $\frac{7\pi}{5}$

Q.12 The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right)+\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $\frac{\pi}{2}$ (b) $\frac{5\pi}{3}$ (c) $\frac{10\pi}{3}$ (d) 0

Q.13 $\sin\left\{2\cos^{-1}\left(\frac{-3}{5}\right)\right\}$ is equal to

(a) $\frac{6}{25}$ (b) $\frac{24}{25}$ (c) $\frac{4}{5}$ (d) $-\frac{24}{25}$

Q.14 If $q = \sin^{-1}\{\sin(-600°)\}$, then one of the possible value of q is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $-\frac{2\pi}{3}$

Q.15. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right)-4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)+2\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\frac{\pi}{3}$ then x is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\frac{\sqrt{3}}{4}$

Q.16 If 4 cos^{-1} x + sin^{-1} x = p, then the value of x is

(a) $\frac{3}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

Q.17 If $\tan^{-1}\frac{x+1}{x-1}+\tan^{-1}\frac{x-1}{x}=\tan^{-1}(-7)$, then the value of x is

(a) 0 (b) -2 (c) 1 (d) 2

Q.18 If sin^{-1} x – cos^{-1} x = p/6, then x =

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{\sqrt{3}}{2}$

Q.19 In a DABC, if C is a right angle, then

$$\tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{b}{c+a}\right)=$$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{2}$ (d) $\frac{\pi}{6}$

Q.20 The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{1}{3\sqrt{3}}$

Q.21 $\cot\left(\frac{\pi}{4}-2\cot^{-1}3\right)=$

(a) 7 (b) 6 (c) 5 (d) none of these

Q.22 $\tan^{-1}(\cot q) = 2q$, then q =

(a) $\pm\frac{\pi}{3}$ (b) $\pm\frac{\pi}{4}$ (c) $\pm\frac{\pi}{6}$ (d) none of these

WORKSHEET-1B

Instructions:

Question 1-10 carry 2 marks each.

Questions 11-20 carry 4 marks each.

	QUESTIONS	ANSWERS
Q1	Find the principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ and $\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$.	$-\frac{\pi}{4}$
Q2	Find the principal values of $\cos^{-1}\frac{\sqrt{3}}{2}$ and $\cos^{-1}\left(-\frac{1}{2}\right)$.	$\frac{2\pi}{3}$
Q3	Find the principal values of $\cot^{-1}\sqrt{3}$ and $\cot^{-1}(-1)$.	$\frac{3\pi}{4}$
Q4	Evaluate each of the following : (i) $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$ (ii) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$ (iii) $\tan^{-1}\left(\tan\frac{\pi}{4}\right)$ (iv) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ (v) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ (vi) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$	(i) $\frac{\pi}{3}$ (ii) $\frac{2\pi}{3}$ (iii) $\frac{\pi}{4}$ (iv) $\frac{\pi}{3}$ (v) $\frac{5\pi}{6}$ (vi) $-\frac{\pi}{4}$

Q5	Prove that : $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$	
Q6	Prove that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$	
Q7	Evaluate the following : (i) $\sin^{-1}(\sin 10)$ (ii) $\sin^{-1}(\sin 5)$ (iii) $\cos^{-1}(\cos 10)$ (iv) $\tan^{-1}\{\tan(-6)\}$	. (i) 3π - 10 (ii) 5 - 2π(iii) 4π - 10 (iv) 2π - 6
Q8	Simplify each of the following : (i) $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), -\frac{\pi}{4}<x<\frac{\pi}{4}$ (ii) $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), \frac{\pi}{4}<x<\frac{5\pi}{4}$	(i) $x+\frac{\pi}{4}$ (ii) $x-\frac{\pi}{4}$
Q9	Prove that : $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)=15$	
Q10	Prove that : (i) $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]=\sqrt{\frac{x^2+1}{x^2+2}}$ (ii) $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]=\sqrt{\frac{x^2+1}{x^2+2}}$	
Q11	If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, prove that sin y = $\tan^2\frac{x}{2}$.	
Q12	If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$, prove that $\frac{x^2}{a^2}-\frac{2xy}{ab}\cos\alpha+\frac{y^2}{b^2}=\sin^2\alpha$	

Q13	Prove that : $\tan^{-1}\frac{1-x}{1+x}-\tan^{-1}\frac{1-y}{1+y}=\sin^{-1}\frac{y-x}{\sqrt{1+x^2}\sqrt{1+y^2}}$	
Q14	Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$	
Q15	Solve the following equations : (i) $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$ (ii) $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$ (iii) $\tan^{-1}\frac{x-1}{x+1}+\tan^{-1}\frac{2x-1}{2x+1}=\tan^{-1}\frac{23}{36}$ (iv) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$	. (i) $\pm\frac{1}{\sqrt{2}}$ (ii) $\frac{1}{6}$ (iii) $\frac{4}{3}$ (iv) $\frac{\pi}{4}$
Q16	Solve the following equations : (i) $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$ (ii) $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ (iii) $\sin[2\cos^{-1}\{\cot(2\tan^{-1}x)\}]=0$	. (i) ±1 (ii) $\frac{1}{2}$ (iii) $1\pm\sqrt{2}$
Q17	If $\cos^{-1}\frac{x}{2}+\cos^{-1}\frac{y}{3}=\alpha,$ then prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$.	
Q18	Solve: (1) $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$ (2) $\sin 6x+\sin^{-1}6\sqrt{3}x=-\frac{\pi}{2}$	(1)$2-\sqrt{3}$ (2) -1/12

Q19	If $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$,then find x	-1
Q20	If $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) \ldots \tan^{-1}\left(\frac{1}{1+n(n+1)}\right) =$ $\tan^{-1}(x)$,then find x	$\boldsymbol{\frac{n}{n+2}}$

<u>**WORKSHEET-2A**</u>

CHAPTER- 3 AND 4

MATRICES AND DETERMINANTS

Instructions:

Question 1-24 carry 1 mark each.

Mark the correct alternative in each of the following:

Q.1 If 5 = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then adj A is

(a) $\begin{bmatrix} -d & -b \\ -c & a \end{bmatrix}$ (b) $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ (c) $\begin{bmatrix} d & b \\ c & a \end{bmatrix}$ (d) $\begin{bmatrix} d & c \\ b & a \end{bmatrix}$

Q.2 If A is a singular matrix, then adj A is

(a) non-singular (b) singular (c) symmetric (d) not defined

Q.3 If A, B are two n × n non-singular matrices, then

(a) AB is non-singular (b) AB is singular

(c) $(AB)^{-1} = A^{-1} B^{-1}$ (d) $(AB)^{-1}$ does not exist

Q.4 If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of | adj A | is

(a) a^{27} (b) a^9 (c) a^6 (d) a^2

Q.5 If A= $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (adj (adj A)) is

(a) 14^4 (b) 14^3 (c) 14^2 (d) 14

Q.6 If B is a non-singular matrix and A is a square matrix, then det (B^{-1} AB) is equal to

(a) Det (A^{-1}) (b) Det (B^{-1}) (c) Det(A) (d) Det (B)

Q.7 For any 2 × 2 matrix, if A (adj A)= $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then | A | is equal to

(a) 20 (b) 100 (c)10 (d) 0

Q.8 If $A^5 = O$ such that $A^n \neq I$ for $1 \leq n \leq 4$, then $(I - A)^{-1}$ equals

(a) A^4 (b) A^3 (c) I+A (d) none of these

Q.10 If A satisfies the equation $x^3 - 5x^2 + 4x + \lambda = 0$, then A^{-1} exists if

(a) $\lambda \neq 1$ (b) $\lambda \neq 2$ (c) $\lambda \neq -1$ (d) $\lambda \neq 0$

Q.11 If for the matrix A, $A^3 = I$, then A^{-1} =

(a) A^2 (b) A^3 (c) A (d) none of these

Q.12 If A and B are square matrices such that $B = - A^{-1} BA$, then (A+B) =

(a) O (b) $A^2 + B^2$ (c) $A^2 + 2AB + B^2$ (d) A + B

Q.13 If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then A^5 =

(a) 5A (b) 10A (c) 16A (d) 32A

Q.14 The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if the value of b is

(a) -3 (b) 3 (c) 0 (d) non-existent

Q.15 If d is the determinant of a square matrix A of order n, then the determinant of its adjoint is

(a) d^n (b) d^{n-1} (c) d^{n+1} (d) d

Q.16 If A is a matrix of order 3 and | A | = 8, then | adjA | =

(a) 1 (b) 2 (c) 2^3 (d) 2^6

Q.17 If $A^2 - A + I = 0$, then the inverse of A is

(a) A^{-2} (b) A + I (c) I - A (d) A - I

Q.18 If A is a square matrix such that $A^2 = I$, then A^{-1} is equal to

(a) A + I (b) A (c) 0 (d) 2A

Q.19 Let A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X be a matrix such that A = BX, then X is equal to

(a) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (d) none of these

Q.20 If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals

(a) 19 (b) 1/19 (c) -19 (d) -1/19

Q.21 If $A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ is orthogonal, then x + y =

(a) 3 (b) 0 (c) -3 (d) 1

Q.22 If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to

(a) $-(3A^2 + 2A + 5)$ (b) $3A^2 + 2A + 5$ (c) $3A^2 - 2A - 5$ (d) none of these

<u>WORKSHEET-2B</u>

Instructions:

Question 1-20 carry 4 mark each.

Q1 If $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, find the determinant of the matrix $A^2 - 2A$.

Q2 Evaluate the determinant $\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ Also, prove that $2 \le \Delta \le 4$.

Q3 For what value of x the matrix $A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$ is singular?

Q4 Without expanding prove that $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$

Q5 Without expanding show that $\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$.

Q6 For any scalar p prove that

$$\Delta = \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x).$$

Q7 Prove that $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix} = x^3$

Q8 Show that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Q9 Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc+bc+ca+ab$$

Q10 Show that $\begin{vmatrix} b^2+c^2 & ab & ac \\ ba & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix} = 4a^2b^2c^2$

Q11 If a, b, c are all positive and are pth, qth and rth terms of a G.P., then show that

$$\Delta = \begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

Q12 Prove that $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{vmatrix} = 4(b+c)(c+a)(a+b)$

Q13 Solve $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

Q14 Find A^{-1}, where $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$. Hence solve the system of equations

$$x + 2y - 3z = -4,\ 2x + 3y + 2z = 2, \quad 3x - 3y - 4z = 11$$

Q15 If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations

$$x + 2y + z = 4, -x + y + z = 0,\ x - 3y + z = 2$$

Q16 Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of

equations :

$$x - y + z = 4, \quad x - 2y - 2x = 9, \qquad 2x + y + 3z = 1$$

Q17 Determine the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ and use it to solve the system of

equations : $x + 3z = 9$, $-x + 2y - 2z = 4$, $2x - 3y + 4z = -3$

Q18 Solve the following system of equations, using matrix method

$$x + 2y + z = 7,\ x + 3z = 11,\ 2x - 3y = 1$$

Q19 Show that $A = \begin{bmatrix} 2 & -3 \\ 3 & 4 \end{bmatrix}$ satisfies the equation $x^2 - 6x + 17 = 0$. Hence, find A^{-1}.

Q20 Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

CHAPTER 5 CONTINUITY AND DIFFERENTIABILITY

WORKSHEET-3A

Instructions:

Question 1-14 carry 1 mark each.

Mark the correct alternative in each of the following:

Q.1. If, $f(x) = \begin{cases} ax^2 + b & , \quad 0 \le x < 1 \\ 4 & , \quad x = 1 \\ x + 3 & , \quad 1 < x \le 2 \end{cases}$

then the value of (a, b) for which f(x) cannot be continuous at x = 1, is

(a) (2, 2) (b) (3, 1) (c) (4, 0) (d) (5, 2)

Q.2 If the function f(x) defined by

$f(x) = \begin{cases} \dfrac{\log(1+3x) - \log(1-2x)}{x}, & x \ne 2 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then k =

(a) 1 (b) 5 (c) -1 (d) none of these

Q.3 The value of k which makes

$f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at x = 0, is

(a) 0 (b) 1 (c) -1 (d) none of these

Q.4 If f(x) = $\begin{cases} \dfrac{1-\cos x}{x\sin x}, & x \neq 0 \\ \dfrac{1}{2}, & x = 0 \end{cases}$ then at x = 0, f(x) is

(a) continuous and differentiable (b) differentiable but not continuous

(c) continuous but not differentiable (d) neither continuous nor differentiable

Q.5 The set of points where the function f(x) given by

f(x)= | x - 3 | cos x is differentiable, is

(a) R (b) R-{3} (c) (0, ¥) (d) none of these

Q.6 Let f(x) = $\begin{cases} 1, & x \leq -1 \\ |x|, & -1 < x < 1 \\ 0, & x \geq 1 \end{cases}$ Then, f is

(a) continuous at x = -1 (b) differentiable at x = -1

(c) everywhere continuous (d) everywhere differentiable

Q.7 If f(x) = $\log_x(\log x)$, then f'(x) at x = e is

(a) 0 (b) 1 (c) 1/e (d) 1/2e

Q.8 If x = a $\cos^3$ q, y = a $\sin^3$ q, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(a) $\tan^2$ q (b) $\sec^2$ q (c) sec q (d) | sec q |

Q.9 If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx} =$

(a) $-\frac{2}{1+x^2}$ (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{2-x^2}$ (d) $\frac{2}{2-x^2}$

Q.10 $\frac{d}{dx}\left\{\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\right\}$ equals

(a) 1/2 (b) -1/2 (c) 1 (d) -1

Q.11 The derivative of $\cos^{-1}(2x^2-1)$ with respect to $\cos^{-1}$ x is

(a) 2 (b) $\frac{1}{2\sqrt{1-x^2}}$ (c) 2/x (d) $1 - x^2$

Q.12 If y = $\log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by

(a) ¥ (b) 1 (c) 0 (d) $\frac{1}{2}$

Q.13 If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)$ = log a then $\frac{dy}{dx}$ is equal to

(a) $\frac{x^2-y^2}{x^2+y^2}$ (b) $\frac{y}{x}$ (c) $\frac{x}{y}$ (d) none of these

Q.14 If $y = \tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) none of these

<u>WORKSHEET-3B</u>

Instructions:

Question 1-6 carry 2 mark each.

Question 6-20 carry 4 mark each.

	QUESTIONS	ANSWERS
Q1	Differentiate the following functions w.r.t. x : (i) $\sin(x^2+1)$ (ii) $e^{\sin x}$ (iii) $\log \sin x$	(i) $2x\cos(x^2+1)$ (ii) $e^{\sin x} \times \cos x$ (iii) cot x
Q2	Differentiate the following functions w.r.t. x : (i) e^{e^x} (ii) $\log_7(\log_7 x)$ (iii) $\log_x 2$	(i) $e^{e^x} \times e^x$ (ii) $\frac{1}{\log_7 x.(\log_e 7)^2}$ (iii) $-\frac{1}{(\log_2 x)^2} \times \frac{1}{x\log_e 2}$
Q3	If $y = \cos^{-1}(2x) + 2\cos^{-1}\sqrt{1-4x^2}, 0 < x < \frac{1}{2}$, find $\frac{dy}{dx}$	$\frac{2}{\sqrt{1-4x^2}}$
Q4	If $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$, find $\frac{dy}{dx}$	$\frac{1}{2\sqrt{1-x^2}}$
Q5	If $y = \cos^{-1}\left\{\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}}\right\}$, find $\frac{dy}{dx}$	$\frac{-1}{\sqrt{1-x^2}}$
Q6	Differentiate the following with respect to x: (i) $\cos^{-1}(\sin x)$ (ii) $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$	. (i) -1 (ii) $\frac{2^{x+1}}{1+4^x}\log 2$
Q7	If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ and x ≠ y, prove that	

	$\frac{dy}{dx} = -\frac{1}{(x+1)^2}$	
Q8	If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1} a$, prove that $\frac{dy}{dx} = \frac{y}{x}$	
Q9	If sin y = x sin (a +y), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$	
Q10	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	
Q11	Find the derivative of $\frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}}$ w.r.t x.	$\frac{\sqrt{x}(x+4)^{3/2}}{(4x-3)^{4/3}}\left\{\frac{1}{2x} + \frac{3}{2(x+4)} - \frac{16}{3(4x-3)}\right\}$
Q12	If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.	
Q13	Differentiate the following functions with respect to x: (i) $x^{x^2-3} + (x-3)^{x^2}$ (ii) $y = x^{\sin x} + (\sin x)^x$	
Q14	Differentiate $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, if $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$	
Q15	Differentiate $\tan^{-1}\left(\frac{1+ax}{1-ax}\right)$ with respect to $\sqrt{1+a^2x^2}$	
Q16	If $y = \log\left\{x + \sqrt{x^2+a^2}\right\}$, prove that $(x^2+a^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$	
Q17	$y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.	
Q18	If x = a cos θ + b sin θ and y = a sin θ - b cos θ, prove that	

	$$y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$	
Q19	If y = sin (sin x), prove that $\frac{d^2y}{dx^2}$ + tan x - $\frac{dy}{dx}$ + y $\cos^2 x$ = 0	
Q20	If $x=\sin\left(\frac{1}{a}\log y\right)$, show that $(1-x^2)y_2-xy_1-a^2y=0$	

<u>WORKSHEET-4A</u>

Chapter - APPLICATIONS OF DERIVATIVES

Instructions:

Question 1-18 carry 1 mark each.

Mark the correct alternative in each of the following:

Q 1. If $V=\frac{4}{3}\pi r^3$, at what rate in cubic units is V increasing when r = 10 and $\frac{dr}{dt}$ = 0.01?

(a) π (b) 4π (c) 40π (d) 4π/3

Q 2. Side of an equilateral triangle expands at the rate of 2 cm/sec. The rate of increase of its area when each side is 10 cm is

(a) $10\sqrt{2}$ cm^2/sec (b) $10\sqrt{3}$ cm^2/sec (c) 10 cm^2/sec (d) 5 cm^2/sec

Q 3. The radius of a sphere is changing at the rate of 0.1 cm/sec. The rate of change of its surface area when the radius is 200 cm is

(a) $8\pi cm^2/sec$ (b) $12\pi cm^2/sec$ (c) $160\pi\ cm^2/sec$ (d) $200\ cm^2/sec$

Q 4. If the function $f(x) = 2x^2 - kx + 5$ is increasing on [1, 2], then k lies in the interval

(a) $(-\infty, 4)$ (b) $(4, \infty)$ (c) $(-\infty, 8)$ (d) $(8, \infty)$

Q 5. The point on the curve $y = x^2 - 3x + 2$ where tangent is perpendicular to $y = x$ is

(a) (0, 2) (b) (1,0) (c) (-1,6) (d) (2, -2)

Q 6. The value of c in Lagrange's mean value theorem for the function $f(x) = x(x - 2)$ when $x \in [1, 2]$ is

(a) 1 (b) 1/2 (c) 2/3 (d) 3/2

Q 7. The value of c in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

(a) 1 (b) -1 (c) 3/2 (d) 1/3

Q 8. The approximate value of $(33)^{1/5}$ is

(a) 2.0125 (b) 2.1 (c) 2.01 (d) none of these

Q 10. The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is

(a) $\frac{1}{14}$ (b) 0.01 (c) $\frac{1}{7}$ (d) none of these

Q 11. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when

(a) $x < 2$ (b) $x > 2$ (c) $x > 3$ (d) $1 < x < 2$

Q 12. If the rate of change of area of a circle is equal to the rate of change of its diameter, then its radius is equal to

(a) $\frac{2}{\pi}$ unit (b) $\frac{1}{\pi}$ unit (c) $\frac{\pi}{2}$ units (d) π units

Q 13. In a sphere the rate of change of volume is

(a) π times the rate of change of radius

(b) surface area times the rate of change of diameter

(c) surface area times the rate of change of radius (d) none of these

Q 14. In a sphere the rate of change of surface area is

(a) 8π times the rate of change of diameter (b) 2π times the rate of change of diameter

(c) 2π times the rate of change of radius (d) 8π times the rate of change of radius

Q 15. The line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$, if the value of m is

(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

Q 16. The normal at the point (1, 1) on the curve $2y + x^2 = 3$ is

(a) $x + y = 0$ (b) $x - y = 0$ (c) $x + y + 1 = 0$ (d) $x - y = 1$

Q 17. The normal to the curve $x^2 = 4y$ passing through (1,2) is

(a) $x + y = 3$ (b) $x - y = 3$ (c) $x + y = 1$ (d) $x - y = 1$

Q 18. The points on the curve $9y^2 = x^3$, where the normal to the curve make equal intercepts with the axes are

(a) $\left(4, \pm\frac{8}{3}\right)$ (b) $\left(4, -\frac{8}{3}\right)$ (c) $\left(4, \pm\frac{3}{8}\right)$ (d) $\left(\pm 4, \frac{3}{8}\right)$

WORKSHEET-4B

Instructions:

Question 1-16 carry 4 mark each.

Question 16-20 carry 6 mark each.

	QUESTIONS	ANSWERS
Q1	A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 7 cm.	Ans. 196 π cm^2
Q2	The total cost C(x) associated with the production of x units of an item is given by: $C(x) = 0.005\ x^3 - 0.02\ x^2 + 30x + 5000$ Find the marginal cost when 3 units are produced, whereby marginal cost we mean the instantaneous rate of change of total cost at any level of output	Ans 30.02
Q3	An air force plane is ascending vertically at the rate of 100 km/h. If the radius of the earth is r km, how fast is the area of the earth, visible from the plane, increasing at 3 minutes after is started ascending ? Given that the visible area A at height h is given by $A = 2\pi r^2 \frac{h}{r+h}$.	Ans $\frac{200\pi r^3}{(r+5)^2}$

Q4	A man 2 meters high, walks at a uniform speed of 6 meters per minute away from a lamp post, 5 meters high. Find the rate at which the length of his shadow increases	Ans 4m
Q5	An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of 3/2 c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.	Ans $\frac{3}{8\pi}\,\text{cm/min}$
Q6	A water tank has the slope of an inverted right circular with its axis vertical and vertex lower most. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.	Ans $\frac{35}{88}\,\text{m/h}$
Q7	Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to the x-axis.	(3/2, –7/2)
Q8	Find the points on the curve $9y^2 = x^3$ where normal to the curve makes equal intercepts with the axes.	(4, 8/3) and (4, -8/3)
Q9	Show that the line $\frac{x}{a}+\frac{y}{b}=1$ touches the curve $y = be^{-x/a}$ at the point where it crosses the y-axis.	

Q10	Find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point, where it cuts x-axis.	Tangent = x – 20y - 7 = 0, normal = 20x + y – 140 = 0
Q11	Find the equation of the tangent line to the curve $y = \sqrt{5x-3}$ -2 which is parallel to the line 4x – 2y + 3 = 0.	Ans 80x – 40y – 103 = 0
Q12	Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2).	Ans x + y – 3 = 0
Q13	Find the intervals in which $f(x) = -x^2 - 2x + 15$ is increasing or decreasing	Increasing = (-∞, -10), decreasing = (-1, ∞)
Q14	Find the intervals in which the function f(x) is (i) increasing, (ii) decreasing : $f(x) = 2x^3 - 9x^2 + 12x + 15$	(i) Increasing = (-∞, 1) ∪ (2, ∞), (ii) decreasing = (1, 2)
Q15	Find the intervals in which the function f given by f(x) = sin x + cos x, $0 \le x \le 2\pi$ is increasing or decreasing	increasing $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ decreasing = $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

Q16	Find the intervals in which $f(x) = 2 \log (x - 2) - x^2 + 4x + 1$ is increasing or decreasing.	increasing = (2, 3) decreasing = $(3, \infty)$
Q17	Find the maximum and the minimum values, if any, of the following functions (i) $f(x) = 3x^2 + 6x + 8, x \in R$ (ii) $f(x) = -\|x - 1\| + 5$ for all $x \in R$ (iii) $f(x) = \sin 3x + 4, x \in (-\pi/2, \pi/2)$ (iv) $f(x) = x^3 + 1$ for all $x \in R$ **[NCERT]** (v) $f(x) = \sin (\sin x)$ for all $x \in R$ (vi) $f(x) = \|x + 3\|$ for all $x \in R$	Ans. (i) min = -1 and max. = does not exist (ii) min = does not exist, max. = 5 (iii) min = 3, max = 5 (iv) min = does not exist, max = does not exist (v) min = -sin 1, max = sin 1 (vi) min = 0, max = does not exist
Q18	Find the points at which the function f given by $f(x) = (x - 2)^4 (x + 1)^3$ has (i) local maxima (ii) local minima (iii) points of inflexion	(i) $\frac{2}{7}$ (ii) 2 (iii) -1
Q19	Find the points of local maxima or local minima if any of the following functions. Also find the local maximum or local minimum values, as the case may be :	

	(i) $f(x) = \sin x + \cos x$, where $0 < x < \frac{\pi}{2}$ (ii) $f(x) = \sin x - \cos x$, where $0 < x < 2\pi$ (iii) $f(x) = \sin 2x$, where $0 < x < \pi$ (iv) $f(x) = 2\cos x + x$, where $0 < x < \pi$ (v) $f(x) = 2\sin x - x$, $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$	
Q20	(i) If the sum of the lengths of the hypotenuse and a side of a right-angled triangles is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$. (ii) A square piece of tin of side 24 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. What should be the side of the square to be cut off so that the volume of the box is maximum ? Also, find this maximum volume. (iii_)Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $\frac{2a}{\sqrt{3}}$. (iv) Show that the volume of the larges cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere. (v) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $\frac{4}{27}\pi h^3 \tan^2 \alpha$. Also, show that height of the cylinder is $\frac{h}{3}$. (vi) An open box with a square base is to be made from a given quantity of cardboard of area c^2 square units. Show	

	that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units. (vii) An open tank with a square base and vertical sides is to be constructed from a metal sheet to hold a given quantity of water. Show that the cost of the material will be least when depth of the tank is half of its width.	