Important snaps by Team PIS Class- XIIth

SUBJECT: MATHEMATICS

BOOK : NCERT

TEACHER: ASHISH SINGH YADAV

Chapter 1 **RELATIONS AND FUNCTIONS**

POINTS TO REMEMBER

- *Reflexive relation R in X is a relation with* $(a, a) \in R\ \forall a \in X$.
- *Symmetric relation R in X is a relation satisfying* $(a, b) \in R$ *implies* $(b, a) \in R$.
- *Transitive relation R in X is a relation satisfying* $(a, b) \in R$ *and* $(b, c) \in R$
- implies that $(a, c) \in R$.
- *Equivalence relation R in X is a relation which is reflexive, symmetric and*
- transitive.
- *Equivalence class [a] containing* $a \in X$ *for an equivalence relation R in X is*
- the subset of X containing all elements *b related to a.*
- A function $f : X \to Y$ *is one-one (or injective) if*
- $f(x1) = f(x2) \Rightarrow x1 = x2\ \forall x1, x2 \in X$.

Chapter 1 **POINTS TO REMEMBER**

- □ A function $f : X \to Y$ is onto (or surjective) if given any $y \in Y$, $\exists x \in X$ such
- that $f(x) = y$.
- □ A function $f : X \to Y$ is one-one and onto (or bijective), if f is both one-one
- and onto.
- □ The composition of functions $f : A \to B$ and $g : B \to C$ is the function
- $gof : A \to C$ given by $gof(x) = g(f(x))$ $\forall x \in A$.
- □ A function $f : X \to Y$ is invertible if $\exists g : Y \to X$ such that $gof = IX$ and
- $fog = IY$.
- □ A function $f : X \to Y$ is invertible if and only if f is one-one and onto.

Chapter-1 **POINTS TO REMEMBER**

- Given a finite set X, a function $f : X \to X$ *is one-one (respectively onto) if and*
- only if *f is onto (respectively one-one). This is the characteristic property of a*
- finite set. This is not true for infinite set
- A ***binary operation*** $*$ ***on a set A is a function*** $*$ ***from*** $A \times A$ ***to A.***
- An element $e \in X$ *is the* ***identity element for binary operation*** $*: X \times X \to X$,
- if $a * e = a = e * a \; \forall a \in X$.
- An element $a \in X$ is ***invertible for binary operation*** $*: X \times X \to X$, ***if***
- there exists $b \in X$ *such that* $a * b = e = b * a$ *where, e is the identity for the*
- binary operation $*$. The element *b is called* ***inverse of a and is denoted by*** a^{-1}.
- An operation $*$ on X is ***commutative if*** $a * b = b * a \; \forall a, b$ ***in X.***
- An operation $*$ on X is ***associative if*** $(a * b) * c = a * (b * c) \; \forall a, b, c$ ***in X.***

Chapter 2 INVERSE TRIGONOMETRIC FUNCTIONS

- The domains and ranges (principal value branches) of inverse trigonometric functions are given in the following table:

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$\mathbf{R} - (-1,1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$\mathbf{R} - (-1, 1)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$y = \tan^{-1} x$	$\mathbf{R}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \cot^{-1} x$	$\mathbf{R}$	$(0, \pi)$

Chapter-2 **POINTS TO REMEMBER**

- $y = \sin^{-1} x \Rightarrow x = \sin y$
- $x = \sin y \Rightarrow y = \sin^{-1} x$
- $\sin(\sin^{-1} x) = x$
- $\sin^{-1}(\sin x) = x$
- $\sin^{-1} \dfrac{1}{x} = \operatorname{cosec}^{-1} x$
- $\cos^{-1}(-x) = \pi - \cos^{-1} x$
- $\cos^{-1} \dfrac{1}{x} = \sec^{-1} x$
- $\cot^{-1}(-x) = \pi - \cot^{-1} x$
- $\tan^{-1} \dfrac{1}{x} = \cot^{-1} x$
- $\sec^{-1}(-x) = \pi - \sec^{-1} x$

Chapter 2 POINTS TO REMEMBER

- $\sin^{-1}(-x) = -\sin^{-1} x$
- $\tan^{-1}(-x) = -\tan^{-1} x$
- $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$
- $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x$
- $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$
- $\text{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$
- $\tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x+y}{1-xy}$
- $2\tan^{-1}x = \tan^{-1} \frac{2x}{1-x^2}$
- $\tan^{-1}x - \tan^{-1}y = \tan^{-1} \frac{x-y}{1+xy}$
- $2\tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$

Chapter- 3 MATRICES

Matrix: It is an ordered rectangular arrangement of numbers (or functions). The numbers (or functions) are called the **elements** of the matrix. Horizontal line of elements is **row** of matrix. Vertical line of elements is **column** of matrix.

Numbers written in the horizontal line form a row of the matrix. Number written in the vertical line form a column of the matrix.

Order of Matrix with 'm' rows and 'n' columns is $m \times n$ *(read as m by n)*.

Chapter- 3 POINTS TO REMEMBER

Types of Matrices

- A **row matrix** has only one row (order: $1\times n$)
- A **column matrix** has only one column (order: $m\times 1$)
- A **square matrix** has number of rows equal to number of columns (order: $m \times m$ or $n \times n$.)
- A **diagonal matrix** is a square matrix with all non-diagonal elements equal to zero and diagonal elements not all zeroes.
- A **scalar matrix** is a diagonal matrix in which all diagonal elements are equal.
- An **identity matrix** is a scalar matrix in which each diagonal element is 1 (unity).
- **A zero matrix** or **null matrix** is the matrix having all elements zero.

Chapter- 3 POINTS TO REMEMBER

- **Equal matrices:** two matrices A = $[a_{ij}]_{m\times n}$ and B = $[\,b_{ij}]\ m \times n$ are equal if

  (a) Both have same order

  (b) $a_{ij} = b_{ij} \forall$ i and j

Operations on matrices

- Two matrices can be added or subtracted, if both have same order.
- If A= $[a_{ij}]_{m\times n}$ and B =$[b_{ij}]_{m\times n}$, then

  A $\pm$ B = $[a_{ij} \pm b_{ij}]_{m\times n}$
- $\lambda A = [\lambda\, a_{ij}]_{m\times n}$ where λ is a scalar
- Two matrices A and B can be multiplied if number of columns in A is equal to number of rows in B.

  If $A = [a_{ij}]_{m\times n}$ and $[b_{jk}]_{n\times p}$

  Then $AB = [c_{ik}]_{m\times p}$ where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

Chapter- 3 POINTS TO REMEMBER

Transpose of a Matrix: Let A be any matrix. Interchange rows and columns of A. The new matrix so obtained is transpose of A donated by A'/A^T.

[order of A = m × n ⇨ order of A' = n × m]

Properties of transpose matrices A and B are:

(i) $(A')' = A$

(ii) $(kA)' = kA'$ (k= constant)

(iii) $(A+B)' = A' + B'$

(iv) $(AB)' = B'A'$

Symmetric Matrix and Skew-Symmetric matrix

A square matrix $A = [a_{ij}]$ is symmetric if $A' = A$i.e. .$a_{ij} = a_{ji}\forall$i and j

A square matrix A= $[a_{ij}]$ is skew-symmetric if $A' = -A$ i.e. a_{ij} = - $a_{ji}\forall$ i and j (All diagonal elements are zero in skew-symmetric matrix)

Chapter- 3 POINTS TO REMEMBER

- A is a symmetric matrix if $A' = A$.
- A is a skew symmetric matrix if $A' = -A$.
- Any square matrix can be represented as the sum of a symmetric and a skew symmetric matrix.
- Elementary operations of a matrix are as follows:
  (i) $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$
  (ii) $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$
  (iii) $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$
- If A and B are two square matrices such that $AB = BA = I$, then B is the inverse matrix of A and is denoted by A^{-1} and A is the inverse of B.
- Inverse of a square matrix, if it exists, is unique.

Chapter- 4 **DETERMINANTS**

- Determinant of a matrix $A = [a_{11}]_{1\times 1}$ is given by $|a_{11}| = a_{11}$
- Determinant of a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{12}\, a_{21}$$

- Determinant of a matrix $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is given by (expanding along R_1)

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Chapter- 4 POINTS TO REMEMBER

For any square matrix A, the |A| satisfy following properties.

- $|A'| = |A|$, where A' = transpose of A.
- If we interchange any two rows (or columns), then sign of determinant changes.
- If any two rows or any two columns are identical or proportional, then value of determinant is zero.
- If we multiply each element of a row or a column of a determinant by constant k, then value of determinant is multiplied by k.
- Multiplying a determinant by k means multiply elements of only one row (or one column) by k.
- If $A = [a_{ij}]_{3\times3}$, then $|k.A| = k^3|A|$
- If elements of a row or a column in a determinant can be expressed as sum of two or more elements, then the given determinant can be expressed as sum of two or more determinants.
- If to each element of a row or a column of a determinant the equimultiples of corresponding elements of other rows or columns are added, then value of determinant remains same.

Chapter- 4 POINTS TO REMEMBER

- Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- Minor of an element a_{ij} of the determinant of matrix A is the determinant obtained by deleting i^{th} row and j^{th} column and denoted by M_{ij}.
- Cofactor of a_{ij} of given by $A_{ij} = (-1)^{i+j} M_{ij}$
- Value of determinant of a matrix A is obtained by sum of product of elements of a row (or a column) with corresponding cofactors. For example,

$$|A| = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}.$$

- If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is zero. For example, $a_{11} A_{21} + a_{12} A_{22} + a_{13} A_{23} = 0$

Chapter-4 POINTS TO REMEMBER

- If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $adj\ A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$, where A_{ij} is cofactor of a_{ij}
- $A\,(adj\ A) = (adj\ A)\,A = |A|\,I$, where A is square matrix of order n.
- A square matrix A is said to be singular or non-singular according as $|A| = 0$ or $|A| \neq 0$.
- If $AB = BA = I$, where B is square matrix, then B is called inverse of A. Also $A^{-1} = B$ or $B^{-1} = A$ and hence $(A^{-1})^{-1} = A$.
- A square matrix A has inverse if and only if A is non-singular.
- $A^{-1} = \frac{1}{|A|}(adj\ A)$
- If $a_1 x + b_1 y + c_1 z = d_1$
  $a_2 x + b_2 y + c_2 z = d_2$
  $a_3 x + b_3 y + c_3 z = d_3$,
  then these equations can be written as $A X = B$, where

  $$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Chapter-5 **CONTINUITY AND DIFFERENTIABILITY**

- A real valued function is ***continuous*** at a point in its domain if the limit of the function at that point equals the value of the function at that point. A function is continuous if it is continuous on the whole of its domain.
- Sum, difference, product and quotient of continuous functions are continuous. i.e., if f and g are continuous functions, then

  $(f \pm g)(x) = f(x) \pm g(x)$ is continuous.

  $(f \cdot g)(x) = f(x) \cdot g(x)$ is continuous.

  $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (wherever $g(x) \neq 0$) is continuous.
- Every differentiable function is continuous, but the converse is not true.

Chapter-5 POINTS TO REMEMBER

- Chain rule is rule to differentiate composites of functions. If $f = v \circ u$, $t = u(x)$ and if both $\frac{dt}{dx}$ and $\frac{dv}{dt}$ exist then

$$\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$$

- Following are some of the standard derivatives (in appropriate domains):

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(\log x) = \frac{1}{x}$$

Chapter-5 POINTS TO REMEMBER

- Logarithmic differentiation is a powerful technique to differentiate functions of the form $f(x) = [u(x)]^{v(x)}$. Here both $f(x)$ and $u(x)$ need to be positive for this technique to make sense.
- ***Rolle's Theorem***: If $f : [a, b] \to \mathbf{R}$ is continuous on $[a, b]$ and differentiable on (a, b) such that $f(a) = f(b)$, then there exists some c in (a, b) such that $f'(c) = 0$.
- ***Mean Value Theorem***: If $f : [a, b] \to \mathbf{R}$ is continuous on $[a, b]$ and differentiable on (a, b). Then there exists some c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Chapter-6 **APPLICATION OF DERIVATIVES**

- If a quantity y varies with another quantity x, satisfying some rule $y = f(x)$, then $\frac{dy}{dx}$ (or $f'(x)$) represents the rate of change of y with respect to x and $\left.\frac{dy}{dx}\right]_{x=x_0}$ (or $f'(x_0)$) represents the rate of change of y with respect to x at $x = x_0$.
- If two variables x and y are varying with respect to another variable t, i.e., if $x = f(t)$ and $y = g(t)$, then by Chain Rule

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}, \quad \text{if } \frac{dx}{dt} \neq 0.$$

- A function f is said to be

(a) increasing on an interval (a, b) if
$x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \leq f(x_2)$ for all $x_1, x_2 \in (a, b)$.

Chapter-6 POINTS TO REMEMBER

Alternatively, if $f'(x) \geq 0$ for each x in (a, b)

(b) decreasing on (a,b) if

$x_1 < x_2$ in $(a, b) \Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in (a, b)$.

Alternatively, if $f'(x) \leq 0$ for each x in (a, b)

- The equation of the tangent at (x_0, y_0) to the curve $y = f(x)$ is given by

$$y - y_0 = \left.\frac{dy}{dx}\right]_{(x_0, y_0)} (x - x_0)$$

- If $\frac{dy}{dx}$ does not exist at the point (x_0, y_0), then the tangent at this point is parallel to the y-axis and its equation is $x = x_0$.
- If tangent to a curve $y = f(x)$ at $x = x_0$ is parallel to x-axis, then $\left.\frac{dy}{dx}\right]_{x=x_0} = 0$.

Chapter-6 POINTS TO REMEMBER

- Equation of the normal to the curve $y = f(x)$ at a point (x_0, y_0) is given by

$$y - y_0 = \frac{-1}{\left.\frac{dy}{dx}\right]_{(x_0, y_0)}}(x - x_0)$$

- If $\frac{dy}{dx}$ at the point (x_0, y_0) is zero, then equation of the normal is $x = x_0$.
- If $\frac{dy}{dx}$ at the point (x_0, y_0) does not exist, then the normal is parallel to x-axis and its equation is $y = y_0$.
- Let $y = f(x)$, Δx be a small increment in x and Δy be the increment in y corresponding to the increment in x, i.e., $\Delta y = f(x + \Delta x) - f(x)$. Then dy given by

$$dy = f'(x)dx \quad \text{or} \quad dy = \left(\frac{dy}{dx}\right)\Delta x\,.$$

is a good approximation of Δy when $dx = \Delta x$ is relatively small and we denote it by $dy \approx \Delta y$.
- A point c in the domain of a function f at which either $f'(c) = 0$ or f is not differentiable is called a *critical point* of f.

Chapter-6 POINTS TO REMEMBER

- ***First Derivative Test*** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then

  (i) If $f'(x)$ changes sign from positive to negative as x increases through c, i.e., if $f'(x) > 0$ at every point sufficiently close to and to the left of c, and $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of *local maxima*.

  (ii) If $f'(x)$ changes sign from negative to positive as x increases through c, i.e., if $f'(x) < 0$ at every point sufficiently close to and to the left of c, and $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of *local minima*.

  (iii) If $f'(x)$ does not change sign as x increases through c, then c is neither a point of local maxima nor a point of local minima. Infact, such a point is called *point of inflexion*.

Chapter-6POINTS TO REMEMBER

- ***Second Derivative Test*** Let f be a function defined on an interval I and $c \in$ I. Let f be twice differentiable at c. Then
  (i) $x = c$ is a point of local maxima if $f'(c) = 0$ and $f''(c) < 0$
  The values $f(c)$ is local maximum value of f.
  (ii) $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$
  In this case, $f(c)$ is local minimum value of f.
  (iii) The test fails if $f'(c) = 0$ and $f''(c) = 0$.
  In this case, we go back to the first derivative test and find whether c is a point of maxima, minima or a point of inflexion.
- Working rule for finding absolute maxima and/or absolute minima

  Step 1: Find all critical points of f in the interval, i.e., find points x where either $f'(x) = 0$ or f is not differentiable.

  Step 2:Take the end points of the interval.

  Step 3: At all these points (listed in Step 1 and 2), calculate the values of f.

  Step 4: Identify the maximum and minimum values of f out of the values calculated in Step 3. This maximum value will be the absolute maximum value of f and the minimum value will be the absolute minimum value of f.

Chapter-7 **INTEGRALS**

- Integration is the inverse process of differentiation. In the differential calculus, we are given a function and we have to find the derivative or differential of this function, but in the integral calculus, we are to find a function whose differential is given. Thus, integration is a process which is the inverse of differentiation.

  Let $\frac{d}{dx}F(x) = f(x)$. Then we write $\int f(x)\,dx = F(x) + C$. These integrals are called indefinite integrals or general integrals, C is called constant of integration. All these integrals differ by a constant.
- From the geometric point of view, an indefinite integral is collection of family of curves, each of which is obtained by translating one of the curves parallel to itself upwards or downwards along the y-axis.

Chapter-7 POINTS TO REMEMBER

- Some properties of indefinite integrals are as follows:

1. $\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$

2. For any real number k, $\int k\, f(x)\, dx = k \int f(x)\, dx$

More generally, if $f_1, f_2, f_3, \ldots, f_n$ are functions and $k_1, k_2, \ldots, k_n$ are real numbers. Then

$$\int [k_1 f_1(x) + k_2 f_2(x) + \ldots + k_n f_n(x)]\, dx$$

$$= k_1 \int f_1(x)\, dx + k_2 \int f_2(x)\, dx + \ldots + k_n \int f_n(x)\, dx$$

Chapter-7 POINTS TO REMEMBER

- **Some standard integrals**

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$. Particularly, $\int dx = x + C$

(ii) $\int \cos x\, dx = \sin x + C$

(iii) $\int \sin x\, dx = -\cos x + C$

(iv) $\int \sec^2 x\, dx = \tan x + C$

(v) $\int \text{cosec}^2 x\, dx = -\cot x + C$

(vi) $\int \sec x \tan x\, dx = \sec x + C$

(vii) $\int \text{cosec}\, x \cot x\, dx = -\text{cosec}\, x + C$

(viii) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

(ix) $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

(x) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

(xi) $\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$

(xii) $\int e^x dx = e^x + C$

(xiii) $\int a^x dx = \frac{a^x}{\log a} + C$

(xiv) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

(xv) $\int \frac{dx}{x\sqrt{x^2-1}} = -\text{cosec}^{-1} x + C$

(xvi) $\int \frac{1}{x} dx = \log|x| + C$

Chapter-7 POINTS TO REMEMBER

1. $$\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b},\ a \neq b$$

2. $$\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$$

3. $$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$$

4. $$\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$$

5. $$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$$

where $x^2 + bx + c$ can not be factorised further.

- **Integration by substitution**

A change in the variable of integration often reduces an integral to one of the fundamental integrals. The method in which we change the variable to some other variable is called the method of substitution. When the integrand involves some trigonometric functions, we use some well known identities to find the integrals. Using substitution technique, we obtain the following standard integrals.

(i) $\int \tan x\, dx = \log|\sec x| + C$ (ii) $\int \cot x\, dx = \log|\sin x| + C$

(iii) $\int \sec x\, dx = \log|\sec x + \tan x| + C$

(iv) $\int \operatorname{cosec} x\, dx = \log|\operatorname{cosec} x - \cot x| + C$

Chapter-7 POINTS TO REMEMBER

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log\left|x + \sqrt{a^2+x^2}\right| + c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x + \sqrt{x^2-a^2}\right| + c$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\log\left|x + \sqrt{a^2+x^2}\right| + c$$

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$$

Chapter-7 POINTS TO REMEMBER

- **First fundamental theorem of integral calculus**

  Let the area function be defined by $A(x) = \int_a^x f(x)\,dx$ for all $x \geq a$, where the function f is assumed to be continuous on $[a, b]$. Then $A'(x) = f(x)$ for all $x \in [a, b]$.

- **Second fundamental theorem of integral calculus**

  Let f be a continuous function of x defined on the closed interval $[a, b]$ and let F be another function such that $\frac{d}{dx}F(x) = f(x)$ for all x in the domain of f, then $\int_a^b f(x)\,dx = [F(x) + C]_a^b = F(b) - F(a)$.

  This is called the definite integral of f over the range $[a, b]$, where a and b are called the limits of integration, a being the lower limit and b the upper limit.

Chapter-8 **APPLICATION OF INTEGRALS**

- The area of the region bounded by the curve $y = f(x)$, x-axis and the lines $x = a$ and $x = b$ $(b > a)$ is given by the formula: $\text{Area} = \int_a^b y\,dx = \int_a^b f(x)\,dx$.
- The area of the region bounded by the curve $x = \phi(y)$, y-axis and the lines $y = c$, $y = d$ is given by the formula: $\text{Area} = \int_c^d x\,dy = \int_c^d \phi(y)\,dy$.

Chapter-8 POINTS TO REMEMBER

- The area of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is given by the formula,

$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx \text{, where, } f(x) \geq g(x) \text{ in } [a, b]$$

- If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$, $a < c < b$, then

$$\text{Area} = \int_a^c [f(x) - g(x)]\,dx + \int_c^b [g(x) - f(x)]\,dx .$$

Chapter-9 **DIFFERENTIAL EQUATIONS**

- **Differential Equation:** Equation containing derivatives of a dependent variable with respect to an independent variable is called differential equation.

- **Order of a Differential Equation:** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.

- **Degree of a Differential Equation:** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.

- **Formation of a Differential Equation:** We differentiate the family of curves as many times as the number of arbitrary constant in the given family of curves. Now eliminate the arbitrary constants from these equations.

After elimination, the equation obtained is differential equation.

Chapter-9 POINTS TO REMEMBER

Solution of Differential Equation

(i) **Variable Separable Method**

$\frac{dy}{dx} = f\ (x, y)$.

We Separate the variables and get

$$f(x)dx = g(y)dy$$

Then $\int f\ (x)dx = \int g\ (y)dy + c$ is the required solutions.

Chapter-9 POINTS TO REMEMBER

(ii) **Homogeneous Differential Equation:** A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where f(x, y) and g(x, y) are both homogeneous functions of the same degree in x and y i.e., of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is called a homogeneous differential equation.

For solving this type of equations we substitute y = vx and then $\frac{dy}{dx} = v + x\frac{dv}{dx}$. The equation can be solved by variables separable method.

A homogeneous differential equation can be of the form $\frac{dx}{dy} = F\left(\frac{x}{y}\right)$

To solve this equation, we substitute x = vy and them $\frac{dx}{dy} = v + y\frac{dv}{dy}$ then the equation can be solved by variable separate method.

Chapter-9 POINTS TO REMEMBER

(iii) **Linear Differential Equation:** An equation of the from $\frac{dy}{dx} + Py = Q$

where *P* and *Q* are constant or functions of *x* only is called a linear differential equation. For finding solution of this type of equations, we find integrating factor (*I.F.*) = $e^{\int P\,dx}$

Solution is *y* (*I.F.*) = $\int Q.(\mathrm{I.F.})dx + c$

Similarly, differential equations of the type $\frac{dx}{dy} + Px = Q$ where P and Q are constants or functions of *y* only can be solved.

Here, I.F. = $e^{\int P dy}$ and the solution is x (I.F.) = $\int$ Q × (I.F.) dy + C

Chapter-10 **VECTOR ALGEBRA**

- Position vector of a point P(x, y, z) is given as $\overline{\text{OP}}(=\vec{r}) = x\hat{i} + y\hat{j} + z\hat{k}$, and its magnitude by $\sqrt{x^2 + y^2 + z^2}$.
- The scalar components of a vector are its direction ratios, and represent its projections along the respective axes.
- The magnitude (r), direction ratios (a, b, c) and direction cosines (l, m, n) of any vector are related as:

$$l = \frac{a}{r}, \quad m = \frac{b}{r}, \quad n = \frac{c}{r}$$

- The vector sum of the three sides of a triangle taken in order is $\vec{0}$.

Chapter-10 POINTS TO REMEMBER

- The vector sum of two coinitial vectors is given by the diagonal of the parallelogram whose adjacent sides are the given vectors.
- The multiplication of a given vector by a scalar λ, changes the magnitude of the vector by the multiple $|\lambda|$, and keeps the direction same (or makes it opposite) according as the value of λ is positive (or negative).
- For a given vector $\vec{a}$, the vector $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$ gives the unit vector in the direction of $\vec{a}$.
- The position vector of a point R dividing a line segment joining the points P and Q whose position vectors are $\vec{a}$ and $\vec{b}$ respectively, in the ratio $m : n$

  (i) internally, is given by $\dfrac{n\vec{a} + m\vec{b}}{m + n}$.

  (ii) externally, is given by $\dfrac{m\vec{b} - n\vec{a}}{m - n}$.

Chapter-10 POINTS TO REMEMBER

- The scalar product of two given vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is defined as

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta .$$

Also, when $\vec{a}\cdot\vec{b}$ is given, the angle 'θ' between the vectors $\vec{a}$ and $\vec{b}$ may be determined by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

- If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then their cross product is given as

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Such that $\vec{a}, \vec{b}, \hat{n}$ form right handed system of coordinate axes.

- If we have two vectors $\vec{a}$ and $\vec{b}$, given in component form as $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and λ any scalar,

Chapter-10 POINTS TO REMEMBER

$$\vec{a}+\vec{b} = (a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}\,;$$

$$\lambda\vec{a} = (\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}\,;$$

$$\vec{a}.\vec{b} = a_1b_1+a_2b_2+a_3b_3\,;$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}.$$

Chapter-11 THREE DIMENSIONAL GEOMETRY

- **Direction cosines of a line** are the cosines of the angles made by the line with the positive directions of the coordinate axes.
- If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.
- Direction cosines of a line joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$$

where $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

- **Direction ratios of a line** are the numbers which are proportional to the direction cosines of a line.
- If l, m, n are the direction cosines and a, b, c are the direction ratios of a line

Chapter-11 POINTS TO REMEMBER

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} ; m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} ; n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

- **Skew lines** are lines in space which are neither parallel nor intersecting. They lie in different planes.
- **Angle between skew lines** is the angle between two intersecting lines drawn from any point (preferably through the origin) parallel to each of the skew lines.
- If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two lines; and θ is the acute angle between the two lines; then

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$$

- If a_1, b_1, c_1 and a_2, b_2, c_2 are the direction ratios of two lines and θ is the acute angle between the two lines; then

$$\cos\theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

- Vector equation of a line that passes through the given point whose position vector is $\vec{a}$ and parallel to a given vector $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$.
- Equation of a line through a point (x_1, y_1, z_1) and having direction cosines l, m, n is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Chapter-11 POINTS TO REMEMBER

- The vector equation of a line which passes through two points whose position vectors are $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.
- Cartesian equation of a line that passes through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1} = \dfrac{z - z_1}{z_2 - z_1}$.
- If θ is the acute angle between $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$, then

$$\cos\theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1|\,|\vec{b}_2|} \right|$$

- If $\dfrac{x - x_1}{l_1} = \dfrac{y - y_1}{m_1} = \dfrac{z - z_1}{n_1}$ and $\dfrac{x - x_2}{l_2} = \dfrac{y - y_2}{m_2} = \dfrac{z - z_2}{n_2}$ are the equations of two lines, then the acute angle between the two lines is given by $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$.

Chapter-11 POINTS TO REMEMBER

- Shortest distance between two skew lines is the line segment perpendicular to both the lines.
- Shortest distance between $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is

$$\left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

- Shortest distance between the lines: $\dfrac{x - x_1}{a_1} = \dfrac{y - y_1}{b_1} = \dfrac{z - z_1}{c_1}$ and

$\dfrac{x - x_2}{a_2} = \dfrac{y - y_2}{b_2} = \dfrac{z - z_2}{c_2}$ is

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

- Distance between parallel lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}$ is

$$\left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right|$$

Chapter-11 POINTS TO REMEMBER

- In the vector form, equation of a plane which is at a distance d from the origin, and $\hat{n}$ is the unit vector normal to the plane through the origin is $\vec{r} \cdot \hat{n} = d$.
- Equation of a plane which is at a distance of d from the origin and the direction cosines of the normal to the plane as l, m, n is $lx + my + nz = d$.
- The equation of a plane through a point whose position vector is $\vec{a}$ and perpendicular to the vector $\vec{N}$ is $(\vec{r} - \vec{a}) . \vec{N} = 0$.
- Equation of a plane perpendicular to a given line with direction ratios A, B, C and passing through a given point (x_1, y_1, z_1) is

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

- Equation of a plane passing through three non collinear points (x_1, y_1, z_1),

Chapter-11 POINTS TO REMEMBER

(x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

- Vector equation of a plane that contains three non collinear points having position vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$
- Equation of a plane that cuts the coordinates axes at $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

- Vector equation of a plane that passes through the intersection of planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$, where λ is any nonzero constant.
- Vector equation of a plane that passes through the intersection of two given planes $A_1 x + B_1 y + C_1 z + D_1 = 0$ and $A_2 x + B_2 y + C_2 z + D_2 = 0$ is $(A_1 x + B_1 y + C_1 z + D_1) + \lambda(A_2 x + B_2 y + C_2 z + D_2) = 0$.
- Two planes $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are coplanar if

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$$

Chapter-11 POINTS TO REMEMBER

- Two planes $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are coplanar if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0.$

- In the vector form, if θ is the angle between the two planes, $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$, then $\theta = \cos^{-1} \dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$.

- The angle ϕ between the line $\vec{r} = \vec{a} + \lambda \vec{b}$ and the plane $\vec{r} \cdot \hat{n} = d$ is

$$\sin \phi = \left| \frac{\vec{b} \cdot \hat{n}}{|\vec{b}||\hat{n}|} \right|$$

Chapter-11 POINTS TO REMEMBER

- The angle θ between the planes $A_1x + B_1y + C_1z + D_1 = 0$ and $A_2x + B_2y + C_2z + D_2 = 0$ is given by

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

- The distance of a point whose position vector is $\vec{a}$ from the plane $\vec{r} \cdot \hat{n} = d$ is $|d - \vec{a} \cdot \hat{n}|$
- The distance from a point (x_1, y_1, z_1) to the plane $Ax + By + Cz + D = 0$ is

$$\left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|.$$

Chapter-12 **LINEAR PROGRAMMING**

- A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum) of a linear function of several variables (called **objective function**) subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear **constraints**). Variables are sometimes called **decision variables** and are **non-negative.**
- A few important linear programming problems are:
  (i) Diet problems
  (ii) Manufacturing problems
  (iii) Transportation problems
- The common region determined by all the constraints including the non-negative constraints $x \geq 0$, $y \geq 0$ of a linear programming problem is called the **feasible region** (or **solution region**) for the problem.
- Points within and on the boundary of the feasible region represent **feasible solutions** of the constraints.

  Any point outside the feasible region is an **infeasible solution.**

Chapter-12 POINTS TO REMEMBER

- Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an **optimal solution.**
- The following Theorems are fundamental in solving linear programming problems:

  Theorem 1 Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

  Theorem 2 Let R be the feasible region for a linear programming problem, and let $Z = ax + by$ be the objective function. If R is **bounded**, then the objective function Z has both a **maximum** and a **minimum** value on R and each of these occurs at a corner point (vertex) of R.
- If the feasible region is unbounded, then a maximum or a minimum may not exist. However, if it exists, it must occur at a corner point of R.

Chapter-12 POINTS TO REMEMBER

- **Corner point method** for solving a linear programming problem. The method comprises of the following steps:
  (i) Find the feasible region of the linear programming problem and determine its corner points (vertices).
  (ii) Evaluate the objective function $Z = ax + by$ at each corner point. Let M and m respectively be the largest and smallest values at these points.
  (iii) If the feasible region is bounded, M and m respectively are the maximum and minimum values of the objective function.

  If the feasible region is unbounded, then
  (i) M is the maximum value of the objective function, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, the objective function has no maximum value.
  (ii) m is the minimum value of the objective function, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, the objective function has no minimum value.
- If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum, then any point on the line segment joining these two points is also an optimal solution of the same type.

Chapter-13 **PROBABILITY**

- The conditional probability of an event E, given the occurrence of the event F is given by $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$, $P(F) \neq 0$
- $0 \leq P(E|F) \leq 1$, $\quad P(E'|F) = 1 - P(E|F)$

  $P((E \cup F)|G) = P(E|G) + P(F|G) - P((E \cap F)|G)$
- $P(E \cap F) = P(E)\, P(F|E)$, $P(E) \neq 0$

  $P(E \cap F) = P(F)\, P(E|F)$, $P(F) \neq 0$
- If E and F are independent, then

  $P(E \cap F) = P(E)\, P(F)$

  $P(E|F) = P(E)$, $P(F) \neq 0$

  $P(F|E) = P(F)$, $P(E) \neq 0$

Chapter-13 POINTS TO REMEMBER

- **Theorem of total probability**

  Let $\{E_1, E_2, ..., E_n\}$ be a partition of a sample space and suppose that each of $E_1, E_2, ..., E_n$ has nonzero probability. Let A be any event associated with S, then

  $P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + ... + P(E_n)\,P(A|E_n)$

- **Bayes' theorem** If $E_1, E_2, ..., E_n$ are events which constitute a partition of sample space S, i.e. $E_1, E_2, ..., E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup ... \cup E_n = S$ and A be any event with nonzero probability, then

$$P(E_i \mid A) = \frac{P(E_i)\,P(A|E_i)}{\sum_{j=1}^{n} P(E_j)\,P(A|E_j)}$$

- A random variable is a real valued function whose domain is the sample space of a random experiment.
- The probability distribution of a random variable X is the system of numbers

X	:	x_1	x_2	...	x_n
P(X)	:	p_1	p_2	...	p_n

where, $p_i > 0,\ \sum_{i=1}^{n} p_i = 1,\ i = 1, 2, ..., n$

Chapter-13 POINTS TO REMEMBER

- Let X be a random variable whose possible values $x_1, x_2, x_3, \ldots, x_n$ occur with probabilities $p_1, p_2, p_3, \ldots p_n$ respectively. The mean of X, denoted by μ, is the number $\sum_{i=1}^{n} x_i p_i$.

  The mean of a random variable X is also called the expectation of X, denoted by E (X).

- Let X be a random variable whose possible values $x_1, x_2, \ldots, x_n$ occur with probabilities $p(x_1), p(x_2), \ldots, p(x_n)$ respectively.

  Let $\mu = E(X)$ be the mean of X. The variance of X, denoted by Var (X) or σ_x^2, is defined as $\sigma_x^2 = \mathrm{Var}(X) = \sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)$

  or equivalently $\sigma_x^2 = E(X - \mu)^2$

  The non-negative number

  $$\sigma_x = \sqrt{\mathrm{Var}(X)} = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p(x_i)}$$

  is called the standard deviation of the random variable X.

Chapter-13 POINTS TO REMEMBER

- Var (X) = E (X^2) – $[E(X)]^2$
- Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions :
  (i) There should be a finite number of trials.
  (ii) The trials should be independent.
  (iii) Each trial has exactly two outcomes : success or failure.
  (iv) The probability of success remains the same in each trial.

For Binomial distribution B (n, p), $P(X = x) = {}^nC_x\, q^{n-x} p^x,\ x = 0, 1, \ldots, n$ $(q = 1 - p)$