Important snaps by Team PIS Class- XIth

SUBJECT: MATHEMATICS

BOOK : NCERT

TEACHER: ASHISH SINGH YADAV

Chapter 1SETS

- A set is a well-defined collection of objects.
- There are two methods of representing a set:

  (a) Roster or Tabular form e.g. natural numbers less than 5 = {1, 2, 3, 4}

  (b) Set-builder form or Rule method e.g.: Vowels in English alphabet= {x : x is a vowel in the English alphabet }

- **Types of sets:**

  (i) Empty set or Null set or void set

  (ii) Finite set

  (iii) Infinite set

  (iv) Singleton set

Georg Cantor (1845-1918)

- **Subset:** A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B, \quad \forall a \in A$. We write it as $A \subseteq B$.

Chapter 1 POINTS TO REMEMBER

- **Equal sets:** Two sets A and B are equal if they have exactly the same elements i.e $A = B$ if $A \subset B$ and $B \subset A$.
- **Power set:** The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. $P(A) = \{ B : B \subset A \}$
- If A is a set with $n(A) = m$ then $n[P(A)] = 2^m$.
- **Equivalent sets:** Two finite sets *A* and *B* are equivalent, if their cardinal numbers are same i.e., $n(A) = n(B)$.

Chapter-1 POINTS TO REMEMBER

- **Proper subset and super set:** If $A \subset B$ then A is called the proper subset of B and B is called the superset of A.

Types of Intervals

Open Interval (a, b) = $\{ x \in R : a < x < b \}$

Closed Interval [a, b] = $\{ x \in R : a \le x \le b \}$

Semi open or Semi closed Interval,

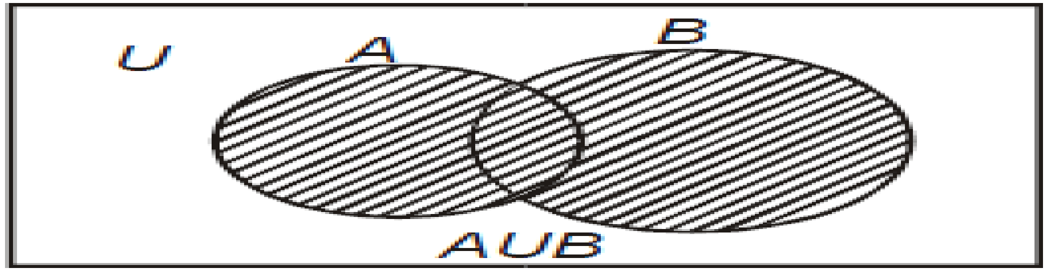
$(a,b] = \{ x \in R : a < x \le b\}$

$[a,b) = \{ x \in R : a \le x < b\}$

Chapter-1 POINTS TO REMEMBER

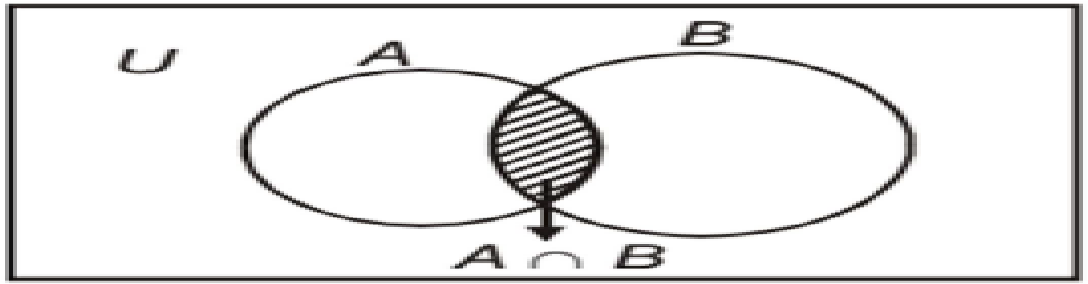
- Union of two sets A and B is,

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$



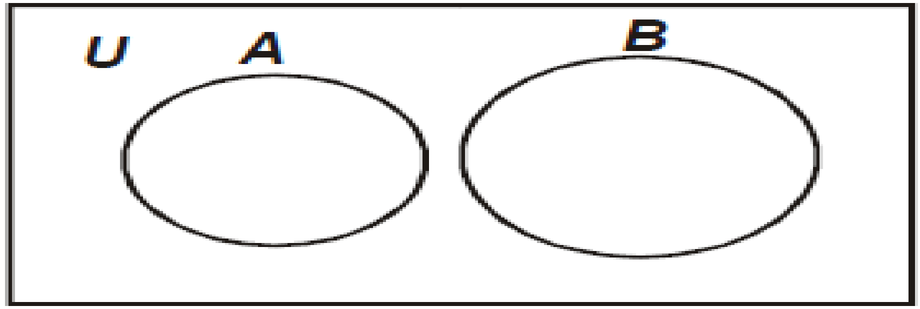
- Intersection of two sets A and B is,

  $A \cap B = \{ x : x \in A \text{ and } x \in B\}$



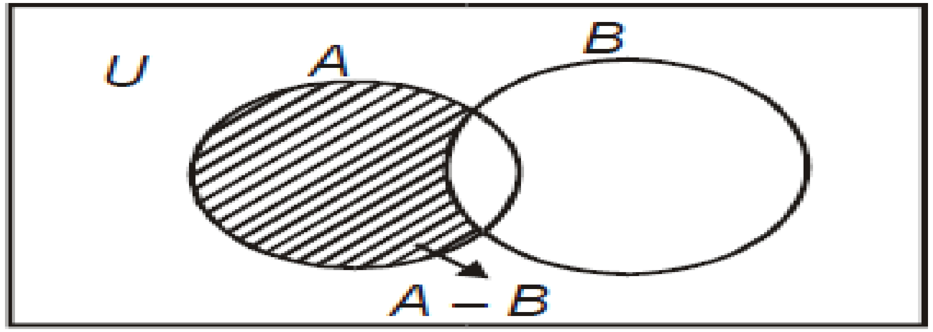
Chapter 1 **POINTS TO REMEMBER**

- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



- Difference of sets A and B is,

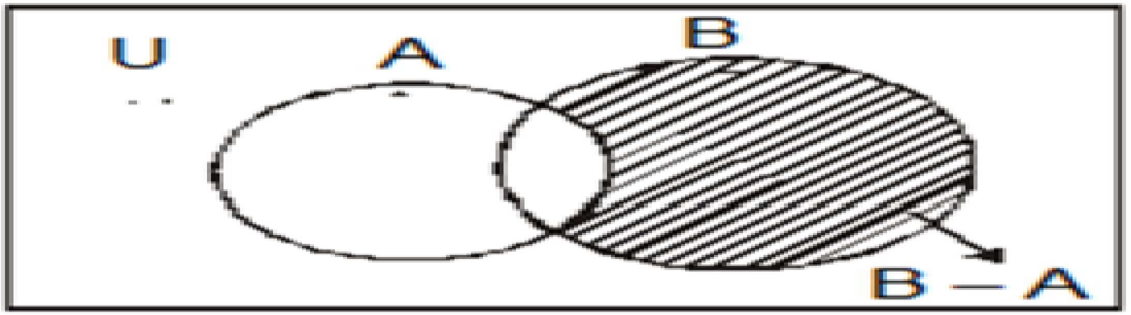
  $A - B = \{ x : x \in A \text{ and } x \notin B\}$



Chapter- 1 POINTS TO REMEMBER

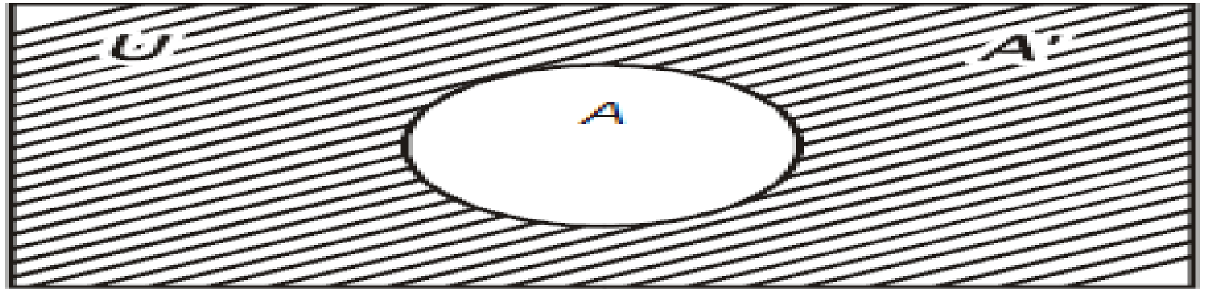
- Difference of sets B and A is,

  $B - A = \{ x : x \in B \text{ and } x \notin A \}$



- Complement of a set A, denoted by A' or A^c is

  $A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



Chapter- 1 POINTS TO REMEMBER

- For any two sets A and B, $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
- If A and B are finite sets such that $A \cap B = \phi$, then

  $n(A \cup B) = n(A) + n(B)$.

  If $A \cap B \neq \phi$, then

  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Chapter- 2 **RELATIONS AND FUNCTIONS**

- ***Ordered pair*** A pair of elements grouped together in a particular order.
- ***Cartesian product*** A × B of two sets A and B is given by

  $A \times B = \{(a, b): a \in A, b \in B\}$

  In particular $\mathbf{R} \times \mathbf{R} = \{(x, y): x, y \in \mathbf{R}\}$

  and $\mathbf{R} \times \mathbf{R} \times \mathbf{R} = (x, y, z): x, y, z \in \mathbf{R}\}$
- If $(a, b) = (x, y)$, then $a = x$ and $b = y$.
- If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.
- $A \times \phi = \phi$
- In general, $A \times B \neq B \times A$.

Chapter- 2 POINTS TO REMEMBER

- ***Relation*** A relation R from a set A to a set B is a subset of the cartesian product A × B obtained by describing a relationship between the first element x and the second element y of the ordered pairs in A × B.
- The ***image*** of an element x under a relation R is given by y, where $(x, y) \in$ R,
- The ***domain*** of R is the set of all first elements of the ordered pairs in a relation R.
- The ***range*** of the relation R is the set of all second elements of the ordered pairs in a relation R.
- ***Function*** A function f from a set A to a set B is a specific type of relation for which every element x of set A has one and only one image y in set B.

  We write f: A⟶B, where $f(x) = y$.
- A is the domain and B is the codomain of f.

Chapter- 2 POINTS TO REMEMBER

- The range of the function is the set of images.
- A real function has the set of real numbers or one of its subsets both as its domain and as its range.
- ***Algebra of functions*** For functions $f : X \to \mathbf{R}$ and $g : X \to \mathbf{R}$, we have

$$(f + g)(x) = f(x) + g(x),\ x \in X$$

$$(f - g)(x) = f(x) - g(x),\ x \in X$$

$$(f.g)(x) = f(x).g(x),\ x \in X$$

$$(kf)(x) = k(f(x)),\ x \in X, \text{ where } k \text{ is a real number.}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ x \in X,\ g(x) \neq 0$$

Chapter- 3 **TRIGONOMETRIC FUNCTIONS**

- 1 radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle.
- π radian = 180 degree, 1° = 60°

$$1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} = 57^{\circ}\ 16'\ 22'' \text{ (Appr.)}$$

- If an arc of length 'ℓ' makes an angle 'θ' radian at the centre of a circle of radius 'r', we have $\theta = \frac{\ell}{r}$.
-

Quadrant →	I	II	III	IV
t-functions which are postive	All	sinx cosecx	tanx cotx	cosx secx

Chapter- 3 POINTS TO REMEMBER

Function	Domain	Range
sinx	R	[–1, 1]
cosx	R	[–1, 1]
tanx	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$; $n \in z$	R
cosecx	$R - \{n\pi\}$; $n \in z$	$R - (-1, 1)$
secx	$R - \left\{(2n+1)\frac{\pi}{2}\right\}$; $n \in z$	$R - (-1, 1)$
cotx	$R - \{n\pi\}$; $n \in z$	R

Chapter-3POINTS TO REMEMBER

- Radian measure $= \frac{\pi}{180} \times$ Degree measure
- Degree measure $= \frac{180}{\pi} \times$ Radian measure
- $\cos^2 x + \sin^2 x = 1$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \operatorname{cosec}^2 x$
- $\cos (2n\pi + x) = \cos x$
- $\sin (2n\pi + x) = \sin x$
- $\sin (-x) = -\sin x$
- $\cos (-x) = \cos x$

Chapter-3POINTS TO REMEMBER

- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$
- $\cos\left(\frac{\pi}{2} - x\right) = \sin x$
- $\sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$ $\quad$ $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos(\pi - x) = -\cos x$ $\quad$ $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$ $\quad$ $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$ $\quad$ $\sin(2\pi - x) = -\sin x$

Chapter-3 POINTS TO REMEMBER

- If none of the angles x, y and $(x \pm y)$ is an odd multiple of $\frac{\pi}{2}$, then

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

- $$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

- If none of the angles x, y and $(x \pm y)$ is a multiple of π, then

$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$

- $$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

- $$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

Chapter-3 POINTS TO REMEMBER

- $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$
- $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$
- $\sin 3x = 3\sin x - 4\sin^3 x$
- $\cos 3x = 4\cos^3 x - 3\cos x$
- $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$
- (i) $\cos x + \cos y = 2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

  (ii) $\cos x - \cos y = -2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

  (iii) $\sin x + \sin y = 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}$

  (iv) $\sin x - \sin y = 2\cos\dfrac{x+y}{2}\sin\dfrac{x-y}{2}$

Chapter-3 POINTS TO REMEMBER

- (i) $2\cos x \cos y = \cos(x+y) + \cos(x-y)$

  (ii) $-2\sin x \sin y = \cos(x+y) - \cos(x-y)$

  (iii) $2\sin x \cos y = \sin(x+y) + \sin(x-y)$

  (iv) $2\cos x \sin y = \sin(x+y) - \sin(x-y)$.
- $\sin x = 0$ gives $x = n\pi$, where $n \in \mathbf{Z}$.
- $\cos x = 0$ gives $x = (2n+1)\dfrac{\pi}{2}$, where $n \in \mathbf{Z}$.
- $\sin x = \sin y$ implies $x = n\pi + (-1)^n y$, where $n \in \mathbf{Z}$.
- $\cos x = \cos y$, implies $x = 2n\pi \pm y$, where $n \in \mathbf{Z}$.
- $\tan x = \tan y$ implies $x = n\pi + y$, where $n \in \mathbf{Z}$.

Chapter-4 Principle of mathematical Induction

Principle of mathematical Induction:

Let $P(n)$ be any statement involving natural number n such that

(i) $P(1)$ is true, and

(ii) If $P(k)$ is true $\Rightarrow P(k + 1)$ is true for some $k \in N$. that is $P(K + 1)$ is true whenever $P(K)$ is true for some $k \in N$ then $P(n)$ is true $\forall\, n \in N$.

Chapter-5 **COMPLEX NUMBERS AND QUADRATIC EQUATIONS**

- A number of the form $a + ib$, where a and b are real numbers, is called a *complex number*, a is called the *real part* and b is called the *imaginary* part of the complex number.
- Let $z_1 = a + ib$ and $z_2 = c + id$. Then
  (i) $z_1 + z_2 = (a + c) + i\,(b + d)$
  (ii) $z_1 z_2 = (ac - bd) + i\,(ad + bc)$
- For any non-zero complex number $z = a + ib$ $(a \neq 0, b \neq 0)$, there exists the complex number $\frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$, denoted by $\frac{1}{z}$ or z^{-1}, called the *multiplicative inverse* of z such that $(a + ib)\left(\frac{a^2}{a^2+b^2} + i\frac{-b}{a^2+b^2}\right) = 1 + i0 = 1$
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$

Chapter-5 POINTS TO REMEMBER

- The conjugate of the complex number $z = a + ib$, denoted by $\bar{z}$, is given by $\bar{z} = a - ib$.
- The polar form of the complex number $z = x + iy$ is $r(\cos\theta + i\sin\theta)$, where $r = \sqrt{x^2 + y^2}$ (the modulus of z) and $\cos\theta = \frac{x}{r}$, $\sin\theta = \frac{y}{r}$. (θ is known as the argument of z. The value of θ, such that $-\pi < \theta \le \pi$, is called the *principal argument* of z.
- A polynomial equation of n degree has n roots.
- The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a, b, c \in \mathbf{R}$, $a \ne 0$, $b^2 - 4ac < 0$, are given by $x = \frac{-b \pm \sqrt{4ac - b^2}\,i}{2a}$.

Chapter-6 **LINEAR INEQUALITIES**

Inequalities: A statement involving '<', '>', '≥' or '≤' is called inequality. Eg., $7 > 5$, $5x - 3 \leq 4$

- Inequalities which do not involve variables are called numerical inequalities.
  Eg., $5 > 9$ and $13 > -2$
- Inequalities which involve variables are called literal inequalities.
  Eg., $3x - 4 \leq 15$ and $4x - 3y \geq 5$
- Inequalities involving the symbols '>' or '<' are called strict inequalities.
- Inequalities involving the symbols '≥' or '≤' are called slack inequalities.

Linear inequalities in one variable: The inequalities of form $ax + b + 0$, $ax + b < 0$, $ax + b \geq 0$ or $ax + b \leq 0$; $a \neq 0$ are called linear inequalities in one variable.

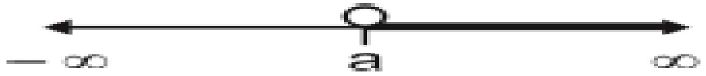
Eg., $4x - 5 \geq 20$ and $-3x - 2 < 5x + 4$

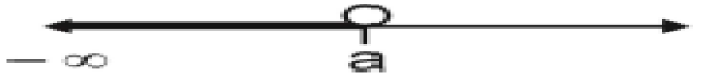
Chapter-6 POINTS TO REMEMBER

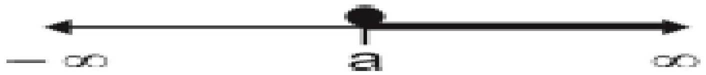
- Equal numbers may be added to (or subtracted from) both sides of an inequality.
- Both sides of an inequality can be multiplied (or divided) by the same positive number. But when both sides are multiplied (or divided) by a negative number, then the inequality is reversed.
- The values of x, which make an inequality a true statement, are called *solutions of the inequality*.
- To represent $x < a$ (or $x > a$) on a number line, put a circle on the number a and dark line to the left (or right) of the number a.
- To represent $x \leq a$ (or $x \geq a$) on a number line, put a dark circle on the number a and dark the line to the left (or right) of the number x.

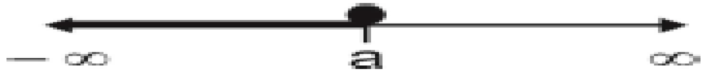
Chapter-6 POINTS TO REMEMBER

- **Graphical representation of solutions on number line:**

(i) $x > a \Leftrightarrow a < x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$ 

(ii) $x > a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$ 

(iii) $x \geq a \Leftrightarrow a \leq x < \infty \Leftrightarrow x \in (a, \infty) \Leftrightarrow$ 

(iv) $x \leq a \Leftrightarrow -\infty < x < a \Leftrightarrow x \in (-\infty, a) \Leftrightarrow$ 

(v) $a < x < b \Leftrightarrow x \in (a, b) \Leftrightarrow$ 

(vi) $a \leq x \leq b \Leftrightarrow x \in (a, b) \Leftrightarrow$ 

- **Linear inequalities in two variables:** The inequalities of form $ax + by + c > 0$, $ax + by + c < 0$, $ax + by + c \geq 0$ or $ax + by + c \leq 0$ are linear inequalities in two variables. $(a, b \neq 0)$

Eg., $4x - 3y < 15$ and $-4x + 15y + 3 \geq 4$

Chapter-6 POINTS TO REMEMBER

Graphical solution of linear inequalities in two variables

- A line divides the Cartesian plane into two parts. Each part is known as a half plane.
- The region containing all the solutions of the inequality is called solution region.
- In order to identify the half plane represented by an inequality (solution region), it is just sufficient to take any point (a, b) not on the line and check whether it satisfy the inequality or not.
- If it satisfies, then the regions containing that point (a, b) is solution region.
- If it does not satisfy, then the other region is solution region.
- If inequality contains '$\geq$' or '$\leq$', then points on line $ax + by = c$ are also included in solution region. In this case we draw dark line while sketching graph of $ax + by = c$.
- If inequality contains '$>$' or '$<$', then points on line $ax + by = c$ are not included in solution region. In this case we draw dotted line while sketching graph of $ax + by = c$.

Chapter-7 **PERMUTATIONS AND COMBINATIONS**

Fundamental principal of counting

- **Multiplication Principle:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is $m \times n$.
- **Fundamental Principle of Addition:** If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in $(m + n)$ ways.

Factorial: Factorial of a natural number n, denoted by n! or n is the continued product of first n natural numbers.

$$n! = n \times (n-1) \times (n-2) \times \dots\dots\dots \times 3 \times 2 \times 1$$

$$= n \times ((n-1)!)$$

$$= n \times (n-1) \times ((n-2)!)$$

Chapter-7 POINTS TO REMEMBER

Permutation: A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.

- The number of permutation of n different objects taken r at a time where $0 \leq r \leq n$ and the objects do not repeat is denoted by ${}^nP_r$ or $P(n, r)$ where,

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

Chapter-7 POINTS TO REMEMBER

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, ... p_k are of k^{th} kind and the rest if any, are of different kinds, is $\dfrac{n!}{p_1!\,p_2!\,.......\,p_k!}$

Combination: Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n objects taken r at a time where,

$0 \le r \le n$, is denoted by ${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$ where ${}^nC_r = \dfrac{n!}{r!(n-r)!}$

Chapter-7 POINTS TO REMEMBER

Some important result:

(i) $0! = 1$

(ii) ${}^nC_0 = {}^nC_n = 1$

(iii) ${}^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

(iv) ${}^nP_r = \lfloor n \; {}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

(v) ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

(vi) If ${}^nC_a = {}^nC_b$ if either $a = b$ or $a + b = n$

Chapter-8 Binomial Theorem

Binomial Theorem for Positive Integers :

- $(x + y)^n = {}^nC_0\, y^0 + {}^nC_1\, x^{n-1} y^1 + {}^nC_2\, x^{n-2} y^2 + \ldots\ldots\ldots\ldots\ldots$

  $\ldots\ldots\ldots\ldots\ldots + {}^nC_r\, x^{n-r} y^r + \ldots\ldots\ldots\ldots\ldots\, {}^nC_2\, x^0 y^n,$

  Where n is any positive integer.

- General Term = $T_{r+1} = {}^nC_r\, x^{n-r} y^r$, where $0 \le r \le n$.

- Total number of terms in expansion $(x + y)^n$ is $n + 1$.

Chapter-8 POINTS TO REMEMBER

Middle Term :

- If n is even, then there is only are middle term

  $$M.T. = \left(\frac{n+1}{2}\right)\text{th term}$$

- If n is odd, then there is two middle terms

  (i) $$M.T. = \left(\frac{n+1}{2}\right)\text{th term}$$

  (ii) $$M.T. = \left(\frac{n+1}{2} + 1\right)\text{th term}$$

Chapter-8POINTS TO REMEMBER

Some important observations :

- In expansion $(x + y)^n$

  T_{r+1} [$(r + 1)^{th}$ term from beginning] = $^nC_r\, x^{n-r}\, y^r$

  T'_{r+1} [$(r + 1)^{th}$ term from end] = $^nC_{n-r}\, x^r\, y^{n-r}$

- $(x + y)^n = {}^nC_0\, x^0 + {}^nC_1\, x^1\, y^1 + {}^nC_2\, x^2\, y^2 + \ldots\ldots\ldots + {}^nC_2\, x^n.$

Chapter-9 **SEQUENCES AND SERIES**

- By a *sequence*, we mean an arrangement of number in definite order according to some rule. Also, we define a sequence as a function whose domain is the set of natural numbers or some subsets of the type $\{1, 2, 3, \ldots k\}$. A sequence containing a finite number of terms is called a *finite sequence*. A sequence is called *infinite* if it is not a finite sequence.
- Let $a_1, a_2, a_3, \ldots$ be the sequence, then the sum expressed as $a_1 + a_2 + a_3 + \ldots$ is called *series*. A series is called *finite series* if it has got finite number of terms.
- An arithmetic progression (A.P.) is a sequence in which terms increase or decrease regularly by the same constant. This constant is called *common difference of the* A.P. Usually, we denote the first term of A.P. by a, the common difference by d and the last term by l. The *general term* or the n^{th} *term* of the A.P. is given by $a_n = a + (n - 1) d$.

  The sum S_n of the first n terms of an A.P. is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l).$$

Chapter-9 POINTS TO REMEMBER

- The *arithmetic mean* A of any two numbers a and b is given by $\frac{a+b}{2}$ i.e., the sequence a, A, b is in A.P.
- A sequence is said to be a *geometric progression* or *G.P.*, if the ratio of any term to its preceding term is same throughout. This constant factor is called the *common ratio*. Usually, we denote the first term of a G.P. by a and its common ratio by r. The general or the n^{th} term of G.P. is given by $a_n = ar^{n-1}$. The sum S_n of the first n terms of G.P. is given by

Chapter-9 POINTS TO REMEMBER

$$S_n = \frac{a(r^n - 1)}{r - 1} \text{ or } \frac{a(1 - r^n)}{1 - r}, \text{ if } r \neq 1$$

The geometric mean (G.M.) of any two positive numbers *a* and *b* is given by

$\sqrt{ab}$ i.e., the sequence *a*, G, *b* is G.P.

Chapter-10 **STRAIGHT LINES**

- *Slope* (m) of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2)

  is given by $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$, $x_1 \neq x_2$.
- If a line makes an angle á with the positive direction of x-axis, then the slope of the line is given by $m = \tan \alpha$, $\alpha \neq 90°$.
- Slope of horizontal line is zero and slope of vertical line is undefined.

Chapter-10 POINTS TO REMEMBER

- An acute angle (say θ) between lines L_1 and L_2 with slopes m_1 and m_2 is given by $\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|, 1 + m_1 m_2 \neq 0$.
- Two lines are *parallel* if and only if their slopes are equal.
- Two lines are *perpendicular* if and only if product of their slopes is -1.
- Three points A, B and C are collinear, if and only if slope of AB = slope of BC.
- Equation of the horizontal line having distance a from the x-axis is either $y = a$ or $y = -a$.
- Equation of the vertical line having distance b from the y-axis is either $x = b$ or $x = -b$.
- The point (x, y) lies on the line with slope m and through the fixed point (x_o, y_o), if and only if its coordinates satisfy the equation $y - y_o = m(x - x_o)$.
- Equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

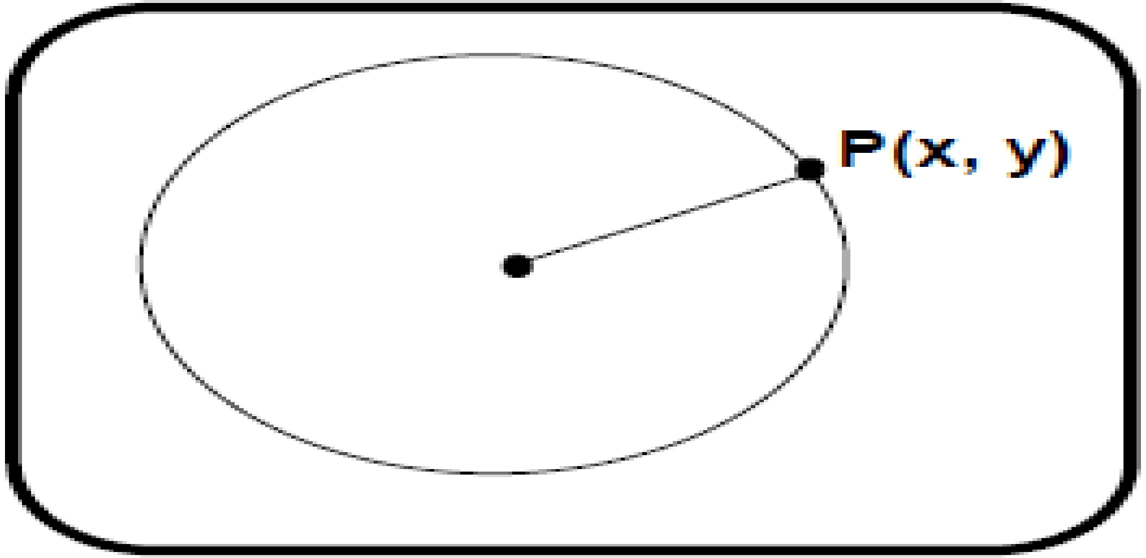
Chapter-10 POINTS TO REMEMBER

- The point (x, y) on the line with slope m and y-intercept c lies on the line if and only if $y = mx + c$.
- If a line with slope m makes x-intercept d. Then equation of the line is $y = m(x - d)$.
- Equation of a line making intercepts a and b on the x-and y-axis, respectively, is $\frac{x}{a} + \frac{y}{b} = 1$.
- The equation of the line having normal distance from origin p and angle between normal and the positive x-axis ω is given by $x \cos \omega + y \sin \omega = p$.
- Any equation of the form $Ax + By + C = 0$, with A and B are not zero, simultaneously, is called the *general linear equation* or *general equation of a line*.
- The perpendicular distance (d) of a line $Ax + By + C = 0$ from a point (x_1, y_1) is given by $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$.
- Distance between the parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$, is given by $d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$.

Chapter-11 **CONIC SECTIONS**

Circle: It is the set of all points in a plane that are equidistant from a fixed point in that plane

Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$ where Centre (h, k), radius $= r$



C (h, k)

CP = CONSTANT = r

Chapter-11 POINTS TO REMEMBER

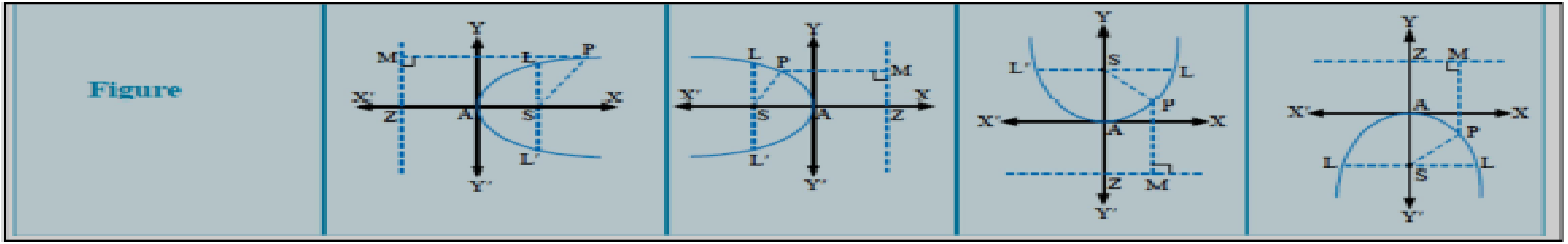
- **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in

	$y^2 = 4ax$ *Parabola towards right*	$y^2 = -4ax$ *Parabola towards left*	$x^2 = 4ay$ *Parabola opening upwards*	$x^2 = -4ay$ *Parabola opening downwards*
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Equation of axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Equation of directrix	$x + a = 0$	$x - a = 0$	$y + a = 0$	$y - a = 0$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$

the plane. Fixed point does not lie on the line

Note: In the standard equation of parabola, a > 0.

$y^2 = 4ax$ $\quad$ $y^2 = -4ax$ $\quad$ $x^2 = 4ay$ $\quad$ $x^2 = -4ay$



Chapter-11 POINTS TO REMEMBER

- An *ellipse* is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.
- The equations of an ellipse with foci on the x-axis is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
- Latus rectum of an ellipse is a line segment perpendicular to the major axis through any of the foci and whose end points lie on the ellipse.
- Length of the latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.
- The eccentricity of an ellipse is the ratio between the distances from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse.
- A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.
- The equation of a hyperbola with foci on the x-axis is : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Chapter-11 POINTS TO REMEMBER

- Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

- Length of the latus rectum of the hyperbola : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is : $\frac{2b^2}{a}$.

- The eccentricity of a hyperbola is the ratio of the distances from the centre of the hyperbola to one of the foci and to one of the vertices of the hyperbola.

Chapter-12 **INTRODUCTION TO THREE DIMENSIONAL GEOMETRY**

- In three dimensions, the coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called the x, y and z-axes.
- The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-*planes*.
- The three coordinate planes divide the space into eight parts known as *octants*.
- The coordinates of a point P in three dimensional geometry is always written in the form of triplet like (x, y, z). Here x, y and z are the distances from the YZ, ZX and XY-planes.
- (i) Any point on x-axis is of the form $(x, 0, 0)$
  (ii) Any point on y-axis is of the form $(0, y, 0)$
  (iii) Any point on z-axis is of the form $(0, 0, z)$.
- Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

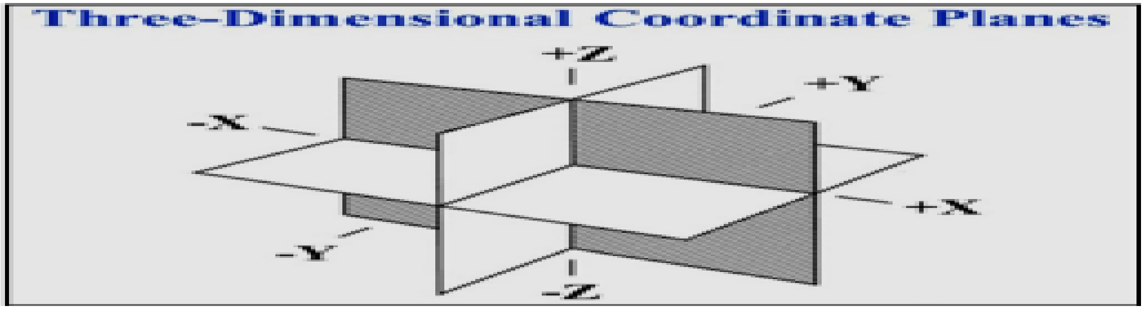
- The coordinates of the point R which divides the line segment joining two points P $(x_1\, y_1\, z_1)$ and Q (x_2, y_2, z_2) internally and externally in the ratio $m : n$ are given by

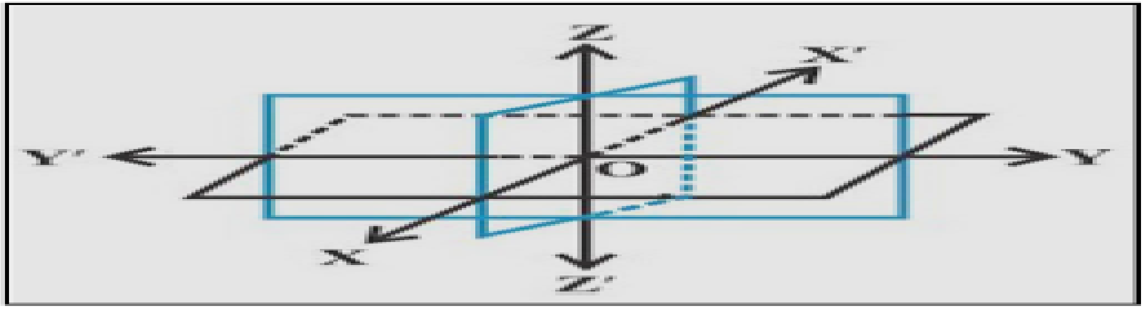
$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right) \text{ and } \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right),$$

respectively.

Chapter-12 POINTS TO REMEMBER

Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.





Coordinate axes: XOX', YOY', ZOZ'

Coordinate planes: XOY, YOZ, ZOX or XY, YX, ZX planes

Octants: OXYZ, OX'YZ, OXY'Z, OXYZ', OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z'

Coordinates of a points lying on x-axis, y-axis and z-axis are of the form (x, 0, 0), (0, y, 0), (0, 0, z) respectively.

Coordinates of a points lying on xy-plane, yz-plane and xz-plane are of the form (x, y, 0), (0, y, z), (x, 0, z) respectively.

Chapter-13 **LIMITS AND DERIVATIVES**

To check whether limit of f(x) as x approaches to exists i.e., $\lim_{x \to c} f(x)$ exists, we proceed as follows.

(i) Find L.H.L at x = a using L.H.L. = $\lim_{h \to 0} f(a-h)$.

(ii) Find R.H.L at x = a using R.H.L. = $\lim_{h \to 0} f(a+h)$.

(iii) If both L.H.L. and R.H.L. are finite and equal, then limit at x = a i.e., $\lim_{x \to a} f(x)$ exists and equals to the value obtained from L.H.L or R.H.L else we say "limit does not exist".

$\lim_{x \to c} f(x) = l$, if and only if $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = l$

$\lim_{x \to c} a = a$, where a is a fixed real number.

$\lim_{x \to c} x^n = c^n$, for all $n \in N$

Chapter-13 POINTS TO REMEMBER

- For a function f and a real number a, $\lim_{x\to a} f(x)$ and $f(a)$ may not be same (In fact, one may be defined and not the other one).
- For functions f and g the following holds:

$$\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a} f(x)\pm\lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x).g(x)]=\lim_{x\to a} f(x).\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

- Following are some of the standard limits

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$

Chapter-13 POINTS TO REMEMBER

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

- The derivative of a function f at a is defined by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

- Derivative of a function f at any point x is defined by

$$f'(x) = \frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- For functions u and v the following holds:

$$(u \pm v)' = u' \pm v'$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$ provided all are defined.

Chapter-13 POINTS TO REMEMBER

SOME IMPORTANT RESULTS ON DERIVATIVE:

- $\frac{d(\sin x)}{dx} = \cos x$
- $\frac{d(\cos x)}{dx} = -\sin x$
- $\frac{d(\tan x)}{dx} = \sec^2 x$

- $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$
- $\frac{d(\sec x)}{dx} = \sec x \cdot \tan x$
- $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cdot \cot x$

- $\frac{d(x^n)}{dx} = n \cdot x^{n-1}$
- $\frac{d(\sqrt{x})}{dx} = \frac{1}{2\sqrt{x}}$
- $\frac{d(a)}{dx} = 0,\ a = \text{constant}$

- $\frac{d(e^x)}{dx} = e^x$
- $\frac{d(\log x)}{dx} = \frac{1}{x}$
- $\frac{d(a^x)}{dx} = a^x \cdot \log a$

Chapter-14 MATHEMATICAL REASONING

- A sentence is called a statement if it is either true or false but not both simultaneously.
- The denial of a statement p is called its negation and is written as ~p and read as not p.
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If–then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.
- Two simple statements p and q connected by the word 'and' namely 'p and q' is called a conjunction of p and q and is written as $p \wedge q$.

Chapter-14 POINTS TO REMEMBER

Compound statement with 'And'

- is true if all its component statements are true
- false if any of its component statement is false

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Two simple statements p and q connected by the word 'or' the resulting compound statement 'p or q' is called disjunction of p and q and is written as $p \vee q$.

Chapter-14 POINTS TO REMEMBER

Compound statement with 'Or' is

- true when at least one component statement is true,
- false when both the component statements are false.

p	q	$p \wedge q$
T	T	T
T	F	T
F	T	T
F	F	F

The negation of the compound statement 'p or q' is '~p and ~q'
$\Rightarrow \sim(p \vee q) = \sim p \wedge \sim q$.

The negation of the compound statement 'p and q' is '~p or ~q'
$\Rightarrow \sim(p \wedge q) = \sim p \vee \sim q$.

A statement with "If p then q" can be rewritten as:-

(a) p implies q

(b) p is sufficient condition for q

(c) q is necessary condition for p

(d) p only if q

(e) (~q) implies (~p)

Chapter-14 POINTS TO REMEMBER

- If in a compound statement containing the connective “or” all the alternatives cannot occur simultaneously, then the connecting word “or” is called as exclusive “or”.
- If, in a compound statement containing the connective “or”, all the alternative can occur simultaneously, then the connecting word “or” is called as inclusive “or”.
- Contrapositive of the statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- Converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$
- “For all”, “For every” are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

Chapter-15 **STATISTICS**

- **Measures of dispersion** Range, Quartile deviation, mean deviation, variance, standard deviation are measures of dispersion.

  Range = Maximum Value – Minimum Value

- **Mean deviation for ungrouped data**

$$\text{M.D.}(\bar{x}) = \frac{\sum \left| x_i - \bar{x} \right|}{n}, \quad \text{M.D. (M)} = \frac{\sum \left| x_i - \text{M} \right|}{n}$$

Chapter-15 POINTS TO REMEMBER

- **Mean deviation for grouped data**

$$\text{M.D.}(\bar{x}) = \frac{\sum f_i (x_i - \bar{x})}{N}, \qquad \text{M.D.}(M) = \frac{\sum f_i (x_i - M)}{N}, \text{ where } N = \sum f_i$$

- **Variance and standard deviation for ungrouped data**

$$\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2, \qquad \sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

- **Variance and standard deviation of a discrete frequency distribution**

$$\sigma^2 = \frac{1}{N} \sum f_i (x_i - \bar{x})^2, \qquad \sigma = \sqrt{\frac{1}{N} \sum f_i (x_i - \bar{x})^2}$$

- **Variance and standard deviation of a continuous frequency distribution**

$$\sigma^2 = \frac{1}{N} \sum f_i (x_i - \bar{x})^2, \qquad \sigma = \frac{1}{N} \sqrt{N \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2}$$

- **Shortcut method to find variance and standard deviation.**

$$\sigma^2 = \frac{h^2}{N^2}\left[N \sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right], \quad \sigma = \frac{h}{N} \sqrt{N \sum f_i y_i^2 - \left(\sum f_i y_i\right)^2},$$

where $y_i = \dfrac{x_i - A}{h}$

- **Coefficient of variation (C.V.)** $= \dfrac{\sigma}{\bar{x}} \times 100, \ \bar{x} \neq 0.$

For series with equal means, the series with lesser standard deviation is more consistent or less scattered.

Chapter-16 **PROBABILITY**

Random Experiment: If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.

Sample Space: The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space (set) is called a sample point.

Chapter-16 POINTS TO REMEMBER

Some examples of random experiments and their sample spaces

(i) A coin is tossed
S = {H, T}, n(S) = 2 Where n(S) is the number of elements in the sample space S.

(ii) A die is thrown
S = { 1, 2, 3, 4, 5, 6], n(S) = 6

(iii) A card is drawn from a pack of 52 cards n (S) = 52.

(iv) Two coins are tossed
S = {HH, HT, TH, TT}, n(S) = 4

(v) Two dice are thrown

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Chapter-16 POINTS TO REMEMBER

- ***Sample points***: Elements of sample space
- ***Event***: A subset of the sample space
- ***Impossible event*** : The empty set
- ***Sure event***: The whole sample space
- ***Complementary event or 'not event'*** : The set A' or $S - A$
- ***Event* A *or* B**: The set $A \cup B$
- ***Event* A *and* B**: The set $A \cap B$
- ***Event* A *and* not B**: The set $A - B$
- ***Mutually exclusive event***: A and B are mutually exclusive if $A \cap B = \phi$
- ***Exhaustive and mutually exclusive events***: Events $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup \ldots \cup E_n = S$ and $E_i \cap E_j = \phi \;\; \forall\, i \neq j$

Chapter-16 POINTS TO REMEMBER

Probability of an Event: For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A and n(S) is number of elements in set S and $0 \le P(A) \le 1$

(a) If A and B are any two events then

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A \text{ and } B)$$

(b) A and B are mutually exclusive events, then

$P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$ for mutually exclusive events)

(c) $P(A) + P(\bar{A}) = 1$ or $P(A) + P(\text{not } A) = 1$

(d) P (Sure event) = P(S) = 1

(e) P (impossible event) = $P(\phi) = 0$

Chapter-16 POINTS TO REMEMBER

- ***Probability***: Number $P(\omega_i)$ associated with sample point ω_i such that

  (i) $0 \le P(\omega_i) \le 1$

  (ii) $\sum P(\omega_i)$ for all $\omega_i \in S = 1$

  (iii) $P(A) = \sum P(\omega_i)$ for all $\omega_i \in A$. The number $P(\omega_i)$ is called *probability of the outcome* ω_i.

Chapter-16 POINTS TO REMEMBER

Addition theorem for three events

Let A, B and C be any three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$