



ITL PUBLIC SCHOOL
Summer Engagement Programme 2023-24
CLASS XII - MATHEMATICS

Chapter – 2 Inverse Trigonometric Functions

1 MARK MCQ

1/2 Marks Questions

- 1) Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$
 - 2) Using principal value, evaluate the following: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.
 - 3) Find the principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$.
 - 4) Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.
 - 5) Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

Chapter – 3 **Matrices**

1 MARK MCQ

- 1) If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in \mathbb{N}$, then A^{4n} equals

(a) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

2) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

- (a) $1 + \alpha^2 + \beta\gamma = 0$ (b) $1 - \alpha^2 + \beta\gamma = 0$ (c) $1 - \alpha^2 - \beta\gamma = 0$ (d) $1 + \alpha^2 - \beta\gamma = 0$

3) If $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$ and $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then AB is equal to

- (a) B (b) nB (c) B^n (d) $A + B$

4) If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then A^n (where $n \in N$) equals

- (a) $\begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & n^2a \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & na \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} n & na \\ 0 & n \end{bmatrix}$

5) If A is a 3×4 matrix and B is a matrix such that $A'B$ and $B'A$ are both defined. Then, B is of the type

- (a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3

1/2 Marks Questions

1. If $\begin{bmatrix} x+3 & 4 \\ y-4 & x+y \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 3 & 9 \end{bmatrix}$, find x and y .

2. If $X_{m \times 3} Y_{p \times 4} = Z_{2 \times b}$, for three matrices X, Y, Z , find the values of m, p and b .

3. If $\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$, find A^{16} .

4. Evaluate: $\begin{bmatrix} \sin^2 \theta & 1 \\ \cot^2 \theta & 0 \end{bmatrix} + \begin{bmatrix} \cos^2 \theta & 0 \\ -\operatorname{cosec}^2 \theta & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

5. Evaluate the following: $[a \ b] \begin{bmatrix} c \\ d \end{bmatrix} + [a \ b \ c \ d] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

6. Show that the elements on the main diagonal of a skew-symmetric matrix are all zeros.

7. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.

8. Show that the matrix $B'AB$ is symmetric or skew-symmetric according as A is symmetric or skew-symmetric.

9. If $f(x) = 3x^2 - 9x + 7$, then for a square matrix A , write $f(A)$.

10. The total number of elements in a matrix represents a prime number. How many possible orders a matrix can have?

4/6 Mark Questions

- Find the values of x, y, z : if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ obeys the law $A'A = I$.
 - If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$.
 - Find X such that, $X \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$
 - If $A = \begin{bmatrix} 1 \\ -5 \\ 7 \end{bmatrix}, B = [3 \ 1 \ -2]$, verify that $(AB)' = B'A'$.
 - If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, show that $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.
 - Express the following matrix as the sum of a symmetric and a skew-symmetric matrix:
 $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 2 \end{bmatrix}$
 - If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, find the value of $A^2 + 2A + 7I$.
 - If $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$, then show that $A^2 = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix}$
 - Find the matrix X such that $\begin{bmatrix} 2 & -1 \\ 0 & 1 \\ -2 & 4 \end{bmatrix} X = \begin{bmatrix} -1 & -8 & -10 \\ 3 & 4 & 0 \\ 10 & 20 & 10 \end{bmatrix}$

Chapter – 4

Determinants

1 Mark MCQ

- 1) If A and B are square matrices of order 2, then $\det(A + B) = 0$ is possible only when

 - (a) $\det(A) = 0$ or $\det(B) = 0$
 - (b) $\det(A) + \det(B) = 0$
 - (c) $\det(A) = 0$ and $\det(B) = 0$
 - (d) $A + B = 0$

2) The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if the value of b is

 - (a) -3
 - (b) 3
 - (c) 0
 - (d) non-existent

3) Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$. If $AX = B$, then X is equal to

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

1/2 Mark Questions

1. If $\begin{vmatrix} x+2 & 3 \\ x+5 & 4 \end{vmatrix} = 3$, the value of x is

2. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.

3. Given determinant $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$. Find the value of $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{33}$.

4. Find the value of p , such that the matrix $\begin{bmatrix} -1 & 2 \\ 4 & p \end{bmatrix}$ is singular.

5. For the two given square matrices A and B of the same order, such that $|A| = 20$ and $|B| = -20$, then $|AB|$

6. Find the inverse of the matrix $\begin{bmatrix} 1 & 3 \\ -6 & -18 \end{bmatrix}$, is possible.

7. If $A = \begin{pmatrix} x & 0 & 1 \\ 2 & -1 & 4 \\ 1 & 2 & 0 \end{pmatrix}$ is a singular matrix, find x .

8. Find the value of x , if area of triangle is 35 square cms with vertices $(x, 4)$, $(2, -6)$ and $(5, 4)$.

9. If for matrix A, $|A| = 3$, find $|5A|$, where matrix A is of order 2×2 .

10. Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, such that $|A| = -10$. Find $a_{11}C_{11} + a_{12}C_{12}$.

11. A is a non-singular matrix of order 3 and $|A| = -4$. Find $|\text{adj. } A|$.

12. Given a square matrix A of order 3×3 , such that $|A| = 12$, find the value of $|A \cdot \text{adj. } A|$.

4/6 Mark Questions

1. Find a matrix A such that $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 3 & -2 \end{bmatrix}$.

2. Given two matrices $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ verify that $BA = 6I$. Use the result to solve the system $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$.
 $x + 2y + z = 7$

3. Using matrices, solve the following system of equations: $x + 3z = 11$
 $2x - 3y = 1$

4. Find a matrix A such that $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5. Using matrix method, solve the following system of equations for x, y and z :

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10$$

$$\frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

6. Using matrix method, solve the following system of equations:

$$2x - y + 2z = 3$$

$$2x + y + z = -1$$

$$x - 3y + 2z = 6$$

7. For a matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the numbers a and b such that $A^2 + aA + bI = O$. Hence find A^{-1} .

Chapter 5 DIFFERENTIATION

1 MARKS MCQ

1. If $x = at^2$, $y = 2$ at, then $\frac{d^2y}{dx^2} =$

- (a) $-\frac{1}{t^2}$ (b) $\frac{1}{2at^3}$ (c) $-\frac{1}{t^3}$ (d) $-\frac{1}{2at^3}$

2. If $f(x) = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1-x^2)f''(x) - xf(x) =$

- (a) 1 (b) -1 (c) 0 (d) none of these

3. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx} =$

- (a) $\frac{4x^3}{1-x^4}$ (b) $-\frac{4x}{1-x^4}$ (c) $\frac{1}{4-x^4}$ (d) $\frac{4x^3}{1-x^4}$

4. If $y = \tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{1}{2}$ (b) 0 (c) 1 (d) none of these

1/2 Mark Questions

- Differentiate $\cos^{-1}\sqrt{x}$ w.r.t x .
- Differentiate w.r.t x : $y = \log_7(\log x)$
- Given $f(0) = -2$, $f'(0) = 3$. Find $h'(0)$ where $h(x) = xf(x)$
- Find dy/dx when $y = \sin x^0$

4/6 Mark Questions

1. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$

2. Differentiate the following w.r.t x: $x^{\sin x} + (\sin x)^{\cos x}$

3. Find $\frac{dy}{dx}$, $y = \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2})$

4. If $y = e^{a\sin^{-1} x}$, $-1 \leq x \leq 1$, then show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$

5. If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \pi/2$

6. If $y = x^x$ show that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$

7. If $x = a \sin 2t$ $(1+\cos 2t)$ and $y = b \cos 2t$ $(1-\cos 2t)$, show that $\left(\frac{dy}{dx} \right) = \frac{b}{a}$ at $t = \pi/4$ and

also find $\left(\frac{d^2y}{dx^2} \right)$ at $t = \pi/4$

8. If $\sqrt{1-x^6} + \sqrt{1-y^6} = a^3(x^3 - y^3)$, prove that $\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}$

9. If $f(x) = \lambda x^2 + \mu x + 12$, $f'(4) = 15$ and $f'(2) = 11$, then find λ and μ

10. If $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$, find $\frac{dy}{dx}$

11. If $y = \log(x + \sqrt{x^2 + 1})$, then prove that $(x^2 + 4) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$

12. If $x = \sin(\frac{1}{a} \log y)$, then show that $(1-x^2)y_2 - xy_1 - a^2 y = 0$

13. If $y = \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$, then show that $\frac{dy}{dx} = -\frac{x}{\sqrt{1-x^4}}$

14. If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

If $y = b \tan^{-1} \left\{ \frac{x}{a} + \tan^{-1} \left(\frac{y}{x} \right) \right\}$, find $\frac{dy}{dx}$

15. Given that $\cos x/2 \cdot \cos x/4 \cdot \cos x/8 \dots = \frac{\sin x}{x}$, prove that

$$\frac{1}{2^2} \sec^2 \frac{x}{2} + \frac{1}{2^4} \sec^2 \frac{x}{4} + \frac{1}{2^6} \sec^2 \frac{x}{8} + \dots = \cos ec^2 x - \frac{1}{x^2}$$

$$\begin{cases} \frac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \pi/2 \\ a, & \text{if } x = \pi/2 \\ \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \pi/2 \end{cases}$$

16. Let $f(x) = \begin{cases} \dots & \text{if } f(x) \text{ be a continuous function at } x = \pi/2, \\ a & \text{and } b. \end{cases}$ find a

17. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & x < 0 \\ a, & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16+\sqrt{x}-4}}, & x > 0 \end{cases}$ For what value of a, is f continuous at x = 0?

18. Determine the value of a,b,c for which the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b\sqrt{x^3}}, & x > 0 \end{cases} \quad \text{may be continuous at } x = 0$$

Chapter 6 Application of Derivatives

1 Mark MCQ

- The radius of a sphere is changing at the rate of 0.1 cm/sec. The rate of change of its surface area when the radius is 200 cm is
 (a) $8\pi\text{cm}^2/\text{sec}$ (b) $12\pi\text{cm}^2/\text{sec}$ (c) $160\pi\text{ cm}^2/\text{sec}$ (d) $200\text{cm}^2/\text{sec}$
- The coordinates of the point on the ellipse $16x^2 + 9y^2 = 400$ where the ordinate decreases at the same rate at which the abscissa increases, are
 (a) $(3, 16/3)$ (b) $(-3, 16/3)$ (c) $(3, -16/3)$ (d) $(3, -3)$
- The function $f(x) = x^9 + 3x^7 + 64$ is increasing on
 (a) \mathbb{R} (b) $(-\infty, 0)$ (c) $(0, \infty)$ (d) \mathbb{R}_0

1/2 Mark Questions

- Find the rate of change of area of a square when its side is increasing at the rate of 2cm/min and the length of the side is 10 cm.
- The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 36x + 5$. Find the marginal revenue when $x = 5$.

4/6 Mark Questions

- Find the intervals in which the function $f(x)$ is increasing or decreasing:
 (1) $f(x) = 2x^3 - 9x^2 + 12x + 15$ (2) $f(x) = x^3 - 12x^2 + 36x + 17$
 (3) $f(x) = \sin x - \cos x, 0 < x < 2\pi$
- Find the interval(s) for which the function $f(x) = \log(2+x) - \frac{2x}{2+x}$ is increasing or decreasing
- Water is running into a conical tank of height 10m and diameter 10m at the top, at a constant rate of $18\text{m}^3/\text{min}$. How fast is the water rising in the tank at any instant?
- Find the intervals in which the function is increasing or decreasing:
 $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$.

5. Find the intervals in which the function is increasing or decreasing:
 $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$.
6. An open box with a square base is to be made out of a given quantity of sheet of area a^2 . Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.
7. A window is in the form of a rectangle above which there is a semicircle. If the perimeter of the window is p cm. Show that the window will allow the maximum possible light only when the radius of the semicircle is $\frac{p}{\pi + 4}$ cm.
8. A large spherical balloon is inflated by pumping in $16m^3/\text{sec}$ of gas.. At the instant when the balloon contains $36\pi m^3$ of gas, how fast is the radius increasing.
9. Find the absolute maximum or minimum of the function given by
 $f(x) = \cos^2 x + \sin x, x \in [0, \pi]$.
10. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is $4r/3$.
11. Show that the volume of greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle 30° is $\frac{4}{81}\pi h^3$.
12. Show that the rectangle of maximum area that can be inscribed in a circle of radius r is a square of side $\sqrt{2}r$.
13. Show that the altitude of a right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

Chapter 7

INDEFINITE INTEGRAL

1 Mark MCQ

1. $\int \frac{x}{4+x^4} dx$ is equal to
 (a) $\frac{1}{4} \tan^{-1} x^2 + C$ (b) $\frac{1}{4} \tan^{-1} \left(\frac{x^2}{2} \right)$ (c) $\frac{1}{2} \tan^{-1} \left(\frac{x^2}{2} \right)$ (d) none of these
2. If $\int \frac{\cos 8x + 1}{\tan 2x - \cot 2x} dx = a \cos 8x + C$, then $a =$
 (a) $-\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{1}{8}$
3. The value of $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$ is
 (a) $2 \cos \sqrt{x} + C$ (b) $\sqrt{\frac{\cos x}{x}} + C$ (c) $\sin \sqrt{x} + C$ (d) $2 \sin \sqrt{x} + C$

4. $\int e^x(1 - \cot x + \cot^2 x)dx =$

(a) $e^x \cot x + C$ (b) $-e^x \cot x + C$ (c) $e^x \operatorname{cosec} x + C$ (d) $-e^x \operatorname{cosec} x + C$

5. $\int \frac{\sin^6 x}{\cos^8 x} dx =$

(a) $\tan 7x + C$ (b) $\frac{\tan^7 x}{7} + C$ (c) $\frac{\tan 7x}{7} + C$ (d) $\sec^7 x + C$

1/2 Mark Questions

1. $\int e^{3 \log x} \cdot x^4 dx$

2. $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$

3. $\int \frac{1}{\sqrt{1-x^2} \sin^{-1} x} dx$

4. $\int \frac{1}{e^x + 1} dx$

5. $\int \frac{dx}{\sin^2 x \cos^2 x}$

6. $\int \sec^4 x \cdot \tan x dx$

7. $\int \frac{e^x}{\sqrt{4 - e^{2x}}} dx$

8. $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

9. $\int \frac{\sec^2(\log x)}{x} dx$

10. $\int \frac{e^x + \sin x}{\sqrt{e^x - \cos x}} dx$

11. $\int \frac{\cos x}{\sin x \log \sin x} dx$

12. $\int \frac{3x^2}{1+x^6} dx$

13. $\int \frac{1 - \cot x}{x + \log \operatorname{cosec} x} dx$

14. $\int \frac{2 \cos x - 3 \sin x}{6 \cos x + 4 \sin x} dx$

15. $\int \frac{\cos 2x - \cos 2h}{\cos x - \cosh} dx$

4/6 Mark Questions

Evaluate the following:

1. $\int 5^{5^x} 5^{5^x} 5^x dx$

2. $\int f'(ax+b)[f(ax+b)]^n dx$

3. $\int \frac{1}{\log x} - \frac{1}{(\log x)^2} dx$

4. $\int \frac{\sqrt{x^2+1}[\log(x^2+1) - 2 \log x]}{x^4} dx$

5. $\int \frac{\sin x + \cos x}{\sin^4 x + \cos^2 x} dx$

6. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

7. $\int \frac{\sin x + \cos x}{\sqrt{9+16\sin 2x}} dx$

8. $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta$